

A New Proposal for a Time-Based Semantics for Modalities in Ockham's Logic

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Abstract. Attempts in the 1970s to outline Ockham's modalities are helpful as they point out the temporal character of Ockham's theory of modalities. However, this important insight has been neglected for several decades. Falessi and Schang (2023) is one of the few recent serious attempts to work out a semantic for Ockham's modalities, resorts to possible worlds semantics and largely disregards temporal aspects. This paper begins with a synopsis of important modern approaches to semantics for Ockham's modalities, and proceeds with a discussion on methodological problems concerning a representation of medieval logic and a clarification of Ockham's modal terms. After that, it examines Falessi and Schang's approach in detail, and assesses it against Ockham's own texts and understandings. In doing so, we identify what is inadequate in Falessi and Schang's reconstruction, enabling a return to Ockham's own texts and with the aid of modern logical tools the development of a new approach to represent Ockham's understanding of modalities. We ultimately arrive at a time-based relational semantics without branching time for the representation of several key features of Ockham's modalities.

1 Modern Approaches to Semantics for Ockham's Modalities

Modern logic requires a formal system for the logical analysis of informal notions derived from natural language, including modal terms such as “necessary”, “possible” belong. ([7, p. 45]) Modal logic studies terms that qualify the truth-conditions of statements, these terms include not only typical modal expressions like necessary or possible, but also epistemic, doxastic and temporal terms. ([14, p. 309]) For modal operators in modern modal logic, mathematical semantic approaches are provided, most notably algebraic semantics, topological semantics and possible world semantics or relational semantics by using Kripke models, depending on how logicians wish to use modal logic. ([3, p. 2]) Algebraic semantics treats modalities as “operators on Boolean algebra”, while relational semantics is based on relational structures containing elements like “possible worlds, moments of time, evidential situations, or states of a computer”, the basic idea is that to assign a formula a truth-value relative to each point of a model. ([14, pp. 309, 325]) A relational semantics for modal logic is intensional, in the sense that certain contexts are included in the description of the truth conditions for modal propositions. For this, a set $\mathbf{W}$ containing relevant contexts of

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evaluation, so that an intensional model is defined as a dyadic pair $\langle \mathbf{W}, \mathbf{a} \rangle$ ¹, in which $\mathbf{W}$ stands for a nonempty set of contexts, while $\mathbf{a}$ stands for an assignment function. In this model, an informal way to capture the semantic of the generic modal operator $\Box$ would be that $\Box A$ is true at w , iff A is true in all contexts w that are related to w in the right way, so a binary relation $\mathbf{R}$ on the set of contexts is introduced. So the semantic model is expanded to a triple $\langle \mathbf{W}, \mathbf{R}, \mathbf{a} \rangle$, in which we can define $\Box$ in a formal way as²

$$\mathbf{a}_w(\Box A) = \top \text{ iff for each } \mathbf{v} \text{ such that } w\mathbf{R}\mathbf{v}, \mathbf{a}_v(A) = \top$$

This framework provides a useful tool for a formal characterizing of modal terms in historical logical theories. However, interpreters of historical logical text and historians of logic should also be careful not to project undue presuppositions onto historical, especially ancient and medieval theories, while remaining aware that genuine insights can be gained in applying modern formal frameworks to these theories. ([30, p. 14])

Modern logicians have long been aware of modal logical elements in Ancient Greek and medieval logic. In the mid-twentieth century, interest in the history of logic began to grow, as modern logicians recognised the significance of historical logical texts for philosophical logic. Arthur Prior is a prominent example, his treatment of modalities in junction with tenses is inspired by Ockham, especially the latter's discussion of future contingency. ([38, p. 144]) Hintikka introduced the "statistical model" which he characterizes as "extensional" for the interpretation of Aristotle's *De Interpretatione* 19a 12-18. ([31, p. 36]; [16, p. 151]) He distinguishes between propositions at a certain time and unqualified propositions in the present tense. Propositions of the latter type refer to event types, not event tokens, thus, unlike the former type, they are temporally unqualified, and can change their truth value over time. Possibility interpreted as "sometimes true," expresses frequency, while necessity is interpreted as "always true". According to Hintikka, Aristotle gives a temporal interpretation of modalities. ([31, pp. 26–27]) Hintikka's student, Simo Knuuttila, follows this interpretation. One of his most famous theses is that in the early medieval period, including Thomas Aquinas, the temporal interpretation of modalities was the paradigm for the semantics of modalities. ([21, p. 209]) Regarding possibilities, people in antiquity and earlier medieval period thought in terms of diachronic alternatives, while in later medieval period, Duns Scotus's invention of simultaneous alternative possibilities provided a new paradigm for the semantics of modalities. Logical possibility

¹Here we are talking about propositional modal logic, for predicate modal logic, then we need to introduce a domain $\mathbf{D}$ for quantification, which leads to a triple $\langle \mathbf{W}, \mathbf{D}, \mathbf{a} \rangle$.

²cf. [13, pp. 63–64]. The formal definition of the modality $\Box$ is on page 64.

is understood independently of ontology as compossibility. Extensional modal theories are criticised by Scotus as actualization at one or some points in time should not be taken as a semantic basis for genuine possibility. By introducing the concept of simultaneous alternatives for a semantic interpretation of possibility, Scotus's theory can be seen as a precursor to our modern possible worlds semantics. According to Knuuttila, Ockham's modal theory shares the same insight that there is an ontological independent interpretation of possibility. It can therefore be captured also by applying modern possible worlds semantics. ([20, pp. 198–199])

Knuuttila accordingly identifies these two general paradigms for semantic interpretation of modal notions in medieval theories as the “diachronic” and “synchronic” model. Although occasionally challenged by scholars of medieval philosophy such as Robert Pasnau and John Marenbon, Knuuttila's thesis has become highly influential. Marenbon disagrees with Knuuttila's narrative that in later medieval era there is such a turn to synchronic understanding of possibility. Marenbon contends that there are two senses of possibility in Scotus, one sense is logical and independent of ontology, but the other sense of possibility is based on divine intellect, and for the second sense of possibility, possible worlds semantics does not apply. ([25, pp. 142–144]) Pasnau, for his part, rejects Knuuttila's and his followers' approaches to a semantic interpretation of nominalist logic, which, in his view, imply an ontological commitment to merely possible beings and, in his own words, “even to a possible-worlds semantics”. ([32, pp. 181–210]) Scepticism of this kind, raised by historians of philosophy, is understandable; however, it is not a sufficient reason to reject the use of possible worlds semantics for interpreting modal terms in medieval logic. As historians of logic, we are not primarily concerned with the ontological status of possible worlds, and even when we refer to “possible worlds”, we do not need to take the set of contexts **W** to be worlds; it can be a set of other entities, such as moments of time. Considering that the truth-value of Ockham's propositions is to be assessed relative to the time-point of their articulation³, a relational semantics is an appropriate approach to formally articulate Ockham's semantics of modalities.

Based on these observations, we shall take into consideration a recent attempt to outline a relational semantics for Ockham's modalities by Davide Falessi and Fabien Schang. ([9]) Various researchers have tried to provide a formal semantic reconstruction of Ockham's logic, but most focus on Ockham's theory of supposition. Calvin Normore's 1975 Toronto dissertation ([27]) contains a formalization of Ockham's

³We will quote *Summa Logicae* as *SL*, chapter number, and his other philosophical works from the collected works titled *Opera Philosophica et Theologica* as *OP*, volume number, page number. Ockham gives a quite simple truth condition for the categorical propositions in present-tense which says that such a proposition is true, if and only if when its subject term and predicate term supposit for the same thing (*SL* II, 2). But in his treatment of past- and future-tense propositions, it becomes obvious that in present-tense propositions, the moment of “now” is implicated. See [11, pp. 86, 107].

supposition theory and a formal description of the syntax of Ockham's modal propositional logic and the term logical rules for conversion and syllogistic inference, in Chapter VIII, "*On Necessity and Future Contingents*", Normore discusses semantics for modalities in Ockham. As he notes, historians of logic during the 1960s and 1970s tend to identify Ockham's necessity with omnitemporality, and possibility with truth at some time, this approach, though have a basis in Ockham's own account, for the modalities appearing in Ockham's writing on future contingents, Normore provides a formal model based on Prior's branching tree structure. ([27, pp. 236–262]) Similarly, Elizabeth Karger's 1976 dissertation ([18]) gives a formal model for defining truth-conditions for assertoric, tensed and modal propositions in Ockham. In this model, she introduces moments of time as its elements. ([18, p. 76]) The work of Normore and Karger work is still highly inspiring today.

More recent studies on medieval modal logic employ possible worlds semantics, even though it may seem an "anachronism". ([28]) With the development of modern modal logic since the 1970s, there are more elegant and practical mathematical frameworks available for the reconstruction of historical logical theories. However, the aspect of time is often not considered in the more contemporary research. Falessi and Schang's paper ([9]) is a typical example, and deserves a more detailed examination, as it is one of the very few recent works that makes a concrete effort to provide a detailed semantics for Ockham's modal logic. By examining it closely, we can determine to what extent it is appropriate to use possible worlds semantics for Ockham's modalities. Based on these findings, we shall propose a new way of formulating a semantics for modalities in Ockham.

2 Linguistic and Logical Dimension of Modalities in Medieval Logic

To clarify the meaning of logical terms in historical logical theories such as those of Aristotle or Ockham, it is worth considering whether historians of logic should first construct a formal language using modern methods to outline the semantics of these historical theories. Before the nineteenth century, logic used a subset of natural language regulated by norms and rules that permitted a certain degree of precision and efficient formal calculation. In this sense, we can speak of formal logic in Aristotle and medieval logicians like Ockham, just as Bochenski discusses a history of formal logic ranging from Aristotle to Frege and Gödel. ([4]) However, logical theories from Ancient Greece or medieval Europe are not strictly axiomatic. They include many elements derived from everyday intuition. Although they contain rules of deduction and inference, these are more ad hoc, intended to resolve specific paradoxes or problems rather than being introduced systematically. Based on historical texts, we can sketch particular deductive laws and rules for historical logical theories, but providing semantics by assigning truth-values to statements within a formal model

is not directly feasible, as ancient and medieval logical theories lack corresponding notions of model. Interpreters must therefore construct a model that might express the ontology upon which past logicians built their theories. Research on semantics in Ockham has focused on his theory of signification and supposition, which addresses the meaning and reference of categorical terms and is closely linked to the debate on universals. Milo Crimi, for example, offers a careful examination of the signification of connotative and oblique terms in Ockham. ([8, pp. 437–470]) These analyses are mostly linguistic, as medieval logicians worked with a regimented natural language and grammar and logic were closely related. In comparison, Catarina Dutihl Novaes adopts a purely logical approach, arguing that Ockham’s supposition theory can be seen as a formal semantics, since his account of truth conditions provides formal algorithms, independent of the interpreter’s subjective skills, that compute the set of possible readings of propositions. ([29], [40, p. 87])

However, as medieval logical theories are intermingled with grammar, we have to first identify logical rules and contents identified by medieval logicians, before we start to construct a formal framework for their modern and formal interpretation. A typical example of intermingling of logic and linguistics is the notion of modality and syncategorema. In linguistics, modality is a semantic category which expresses concepts like “necessity”, “possibility”, “obligation” and so on. Being a semantic category allows a cross-linguistic description of modality. Modal logic can help define modality. But linguistically, modality is regarded as an expression of a speaker’s attitude, or an expression outside the proposition, or an expression of factuality distinctions. ([26, pp. 166–168]) Linguists like Charles Bally distinguishes between the subjective and objective content of a sentence, referring to the latter with the term “dictum” and to the former with the term “modus”. Fillmore’s conception of proposition as “tenseless set of relationship involving verbs and nouns” corresponds roughly to what Bally understands as dictum. According to Fillmore, modality is an addition to the proposition, together, they form a sentence. ([26, p. 177], [10, p. 4]) This is a grammatical definition of modality which relies heavily on particular languages and therefore not suitable for a cross-linguistic capture of modality. ([26, p. 179]) However, medieval logicians, as they uniformly used Latin as a standard scientific language, did not face problem like that. The problem is rather, for early medieval logic, linguistic moments are often taken over. When talking about modalities, they refer often to grammatical structures. This leaves also traces in Ockham’s treatment of modality. As he writes that a proposition is modal, if a modal term is added to this proposition. But then he adds that only a modal term that can be predicated of a proposition is sufficient to make a proposition a modal proposition. ([42])

Despite all this, Ockham is aware of fundamental difference between linguistic and logic. Linguistically, modalities can be adverbs, adjectives or verbs. But on the

logical level, Ockham talks about modal propositions *in sensu divisionis* or *in sensu compositionis*. Whether a proposition is in the divided or composite sense, depends on the interpretation of logicians and not on the linguistic structure of a proposition. Therefore, a modal proposition where the modal term appears like a predicate of the *dictum* can be interpreted in the divided sense. A modal proposition in the composite sense is a proposition, in which the modal term is said truly of *dictum* it contains. For example, the proposition “It is necessary that every human being is an animal” (*omnem hominem esse risibilem est necessarium*) says just as much as that the proposition “every human being is an animal” is necessary. The *dictum* contained in this modal proposition is expressed in English in a that-clause, however, in Latin, the *dictum* is expressed in the so-called ACI-construction which contains an infinitive verb, with the subject standing in the accusative case. However, a modal proposition with the same linguistic structure can be interpreted as in the divided sense too. In his *Commentary on Aristotle's On Interpretation*, Ockham does not explain further about the logical behavior of modal propositions in the divided sense though, as according to him, Aristotle is dealing with modal propositions in the composite sense, and quite obviously, the composite sense is equivalent to what we now call the *de dicto* reading.⁴ Later, in his comprehensive logical work *Summa Logicae*, while retaining what is said about the composite sense in his *Commentary*, Ockham gives a logical analysis of modal propositions in the divided sense. Modality in the divided sense concerns the way and proper form in which the predicate term supposits for that which the subject term supposits. (*SL* II, 10, *OP* I, 276, [12, p. 112]) So far we have seen that Ockham is clearly aware of the difference between linguistics and logic, and he gives a logical description of modal propositions in the divided and composite sense. We will give them a more detailed treatment in section 5.

3 Modalities and Syncategoremata in Ockham's Logic

Modal terms and modal logic had been part of medieval logic since Boethius, however, modalities became interrelated to the study of *syncategoremata* which started to emerge in the 13th century. The notion of *syncategoremata* can be traced back to Aristotle's and the grammarian Priscian. In *On Interpretation*, Aristotle points out the special status of the copula, while noun and verbs are normally referring in a way to some things, the copula has no such reference. (*De Int.* 16a19–25) Priscian explains that according to the Aristotelians, nouns and verbs have their own signification, whereas *syncategoremata* only “consignify”. (*Inst. Gramm.* II, 15, 54, [36, pp. 104–105]) However, most 13th century logicians spoke of a kind of meaning or signification of *syncategoremata*, which they took to be one of the properties of term,

⁴Ockham, *Exp. in libr. Perih.* II, 5, *OP* II, 465.

this position presumes that meaning of words is not to be accounted for by reference to things only, in Peter of Spain, they indicate a kind of quality pertaining to the subject or predicate. ([36, p. 107])

With the emergence of the *syncategoremata* treatises, medieval logicians started to treat modal terms within the context of syncategoremata. William of Sherwood discusses “necessary” and “contingent” in his *Syncategoremata*, 15. He explains that “necessary” can be used either as a categorematic or a syncategorematic word, when used categorematically, it is a determination of a predicate, when used syncategorematically, it is a determination of a composition. ([43, p. 100]) For other authors, modal terms like *necessario* and *contingenter* are also counted among *syncategoremata*. ([36, p. 110])

Ockham shared the common view of *syncategoremata* that they don’t refer to things on their own. Conjunctions, expressing logical connectives in natural language, and prepositions belong to *syncategoremata*. *Syncategoremata* can be nouns or adjectives, but they are not terms which can serve as major, minor or middle term in a syllogism, in the sense of Aristotle’s usage of the word in his *Prior Analytics*. (*SL* I, 2, *OP* I, 10, [24, p. 51]) Ockham mentions in *SL* I, 2 several examples of syncategoremata (*SL* I, 4, *OP* I, 15, [24, p. 55]), which are far from an exhaustive list. In *SL* III, 4, Ockham mentions “*necessario*” as a syncategorema. (*SL* III-4, 2, *OP* I, 753) Logically regarded, modal terms meet well all the characteristics of syncategoremata (*SL* III-1, 4, *OP* I, 376). As Rivero shows, syncategorematic terms can function as

modifications of the subject, or as modifications of the predicate, or as modifications of the relation between subject and predicate. Modal terms can play all these roles. They are closely connected with syncategorematic terms because they appear often in Latin as adverbials, that is, as modifications, and not as subject or predicate. In those cases in which they appear as adjectives, that is, as categorematic terms or predicates, Schoolmen considered that they had the same properties as syncategorematic terms. ([34, p. 137])

Ockham counts more syncategoremata as being modal than the traditional four Aristotelian modalities. For him, the sentence “*Omne hominem currere est scitum*” (it is known that all human beings run) and “*omne hominem risibilem est demonstrabile*” (it is demonstrable that all human beings are capable of laughing) to be modal sentences, then *syncategoremata* like *scitum* and *demonstrabile* seem also to belong to modalities. (*OP* II, 243) In the *SL* he makes clear that although Aristotle doesn’t treat more than these four conventional modalities like necessity, impossibility, contingency and possibility, but modal terms are not restricted just to these four, terms like true, false, spoken, written, thought, believed etc. can all form modal sentences.

In *SL* III-1, 41, in the part on modal syllogistic, he treats mixed modal syllogisms containing these modalities in the expanded sense. (*SL* II, 1, *OP* I, 243; *SL* III-1, 41, *OP* I, 463) Ockham seems to have realized that those *syncategoremata* are sentential functors that behave logically differently than those *syncategoremata* that are truth-functional. As he writes that the logic term “modal sentence” is to be understood in two different ways, i. e. taken strictly or freely: Freely spoken, a modal sentence is a sentence containing a modal term or a *modus*. Strictly spoken, a sentence is modal only when the modal term is posited in the way that it can only belong to the sentence alone. In this case the modal term serves as a predicate of the whole sentence, and more precisely can be truly predicated of (*verificabilis*) that sentence. However, grammatically, the modal term doesn't need to be in the adjective form, as a predicate of the sentence, it can also stand in the form of an adverb. (*Exp. in libr. Perih.* II, 5; *OP* II, 463 line 85–93; *SL* II-1, *OP* I, 242–243) It is clear that both modal sentences in the composite sense and in the divided sense are according to Ockham in the proper sense modal sentences. There is strong evidence that Ockham was aware of a sort of an intensional logic. In Ockham's logic, what we would call epistemic logic or deontological logic belong to modal logic.

4 Critical Assessment of Falessi and Schang's Reconstruction of a Formal Semantics for Ockham's Modal Logic

4.1 An informal sketch of Falessi and Schang's reconstruction

Falessi and Schang base their reconstruction of a semantics for modalities in Ockham's logic on a set-theoretical semantics and a second-order logic in which modalities are interpreted as “a dyadic predicate including properties and worlds”. ([9, p. 1]) They set up a semantic model that includes individuals, worlds and properties. If the number of these three types of entities are finite, the number of combinations of the alternatives of each individual having or not having each particular property is also finite and can be enumerated. Modalities can be semantically captured by assigning a certain set of afore mentioned combinations as the truth condition for a certain modal statement. The modalities taken into consideration are all alethic modalities. In difference to standard approaches to modalities, they focus their research on “contingent” as an operator, although they are aware that this modality is defined as “the conjunction of opposite simultaneous possibilities”. ([9, p. 7]) Their reason of taking “contingent” as a starting point is that “necessity” can be defined as being incompatible with contingency⁵, and therefore, by introducing the modality “contingent”, they are able to expand the modal square of oppositions between cate-

⁵They understand “contingent” here as two-sided possibility. Aristotle and Ockham both explain the two senses of “contingent”.

gorical propositions of traditional Aristotelian quantity and quality of copula in form of A, E, I, O on necessity with one categorical propositions in form of SiP⁶ on contingency *de dicto*, and another one with the modality formed by the negation of the modality “contingent” in form of “SiP” *de re* into a hexagon diagram of oppositions. ([9, pp. 8–9])

Falessi and Schang give a formal description of this procedure but their idea can be captured also in an informal way, without having to enumerate all the formal framework and definitions they use in their paper. What is notable here is that their formal language bases on the four types of Aristotelian categorical statement A, E, I, O, correspondently, they have four types of modal propositions *de dicto*, symbolized as “ $\Box A$ ”, taking necessity as an example, and another four types of modal propositions *de re*, symbolized as “ $A\Box$ ”, with the modal operator behind the A, E, I, O symbol. The symbol “ P ” which is used in term logic for predicate term is used by them at the same time as “a dyadic relation between an individual and a world”, and can be symbolized as Paw which reads “(the individual) a is P at w ”. ([9, p. 12]) In their framework, they quantify over possible worlds and objects (individuals), while properties are regarded as relations. They set up a finite model so that truth-value of modal propositions can be described as “knowing which individuals satisfy the property P in which possible world”. ([9, p. 13]) In a finite model, this knowledge is provided by mathematical combinatoric. As there is a finite set of models listing the satisfying and non-satisfying relation of all individuals and worlds. In a minimal model in which there are one property, two individuals and two worlds, 16 models can be enumerated. A semantics for modal propositions can be given by finding out which set of these 16 models belong to the truth-condition of modal propositions.

This “numbering semantics” provided by Falessi and Schang has the advantage that it is computable. However, it bases on abstract entities that do not match objects of the extra-mental physical world which Ockham’s ontology is meant to capture. Ockham’s ontology is based on physical individuals whose existence is limited and continuous in time. Setting up a model with individuals not defined in time cannot explain properly Ockham’s modal logic. Ockham ascribes modalities also to tensed propositions, as true propositions in past-tense are necessary, while some propositions in future-tense are contingent. Falessi and Schang’s model includes property P as a relation between individuals and worlds, but it does not stipulate any relation between the contexts. Their reconstruction is an interesting and worthy attempt, but is not yet satisfactory enough if the intention of a formalized characterization of a semantic is to understand Ockham’s own modal logic properly and better. Therefore, while examining their attempt critically, in the following passages we will try to found out

⁶These are standard forms of Aristotelian term logic that can be found in every basic introduction to logic and therefore will not be explained extra in this paper.

things which are important but missed by them.

4.2 Problems with Falessi and Schang's reconstruction

Falessi and Schang captures modalities *de dicto* and *de re* according to the usual approach in modern modal logic, that is, to treat modal operators in the *de dicto* sense as outside the scope of the quantifier ranging over the domain of individuals, while treating modal operators in the *de re* sense as falling inside the scope of the quantifier over individuals. As they present modal operators as a first kind of quantification over worlds, quantification over individuals becomes a second kind of quantification. The formal structure of a modal proposition *de dicto* is therefore represented as putting the quantification over worlds before the quantification over individuals, while the formal structure of a modal proposition *de re* has the second kind of quantification before the quantification over worlds. ([9, p. 13]) Their reconstruction of the modal square of oppositions described in Ockham's commentary on Aristotle's *De Interpretatione* is represented as a diagram of oppositional relation between modal propositions *de re*. However, the expanded diagram of oppositions of modal propositions given by Falessi and Schang is a representation of oppositional relations between propositions on necessity and possibility *de re* and propositions on contingency *de dicto*. This particularity is due to the reason that Falessi and Schang interpret universal affirmative propositions on contingency in the *de dicto* sense as equivalent to the conjunction of universal affirmative propositions on possibility with mutually exclusive predicate terms, namely: $\Box \Diamond_c \forall x (Sx \rightarrow Px) = \Diamond \forall x (Sx \rightarrow Px) \wedge \Diamond \forall x (Sx \rightarrow \neg Px)$, they also tacitly accept $\Diamond_c \forall x (Sx \rightarrow Px) = \forall x (Sx \rightarrow \Diamond Px) \wedge \forall x (Sx \rightarrow \Diamond \neg Px)$ ⁷, thus turning contingency *de dicto* into *de re*.

But there are several serious problems with Falessi and Schang's interpretation. First of all, Ockham's contingency *de dicto* cannot be interpreted in the way displayed above. For Ockham, however, a proposition on contingency *de dicto* can be defined by "neither necessary nor impossible". (*SL* II, 27, [12, p. 172]) As Wolfgang Lenzen has already pointed out, Ockham has a propositional modal logic ([23, pp. 125–169]), Ockham's modalities *de dicto* or in his own expression, *in sensu compositionis*. refer to propositions, so that we do not need to use Aristotelian term logical notions or formulations while dealing with Ockham's modalities *de dicto*. We can formulate Ockham's definition for contingency *de dicto* as $\Diamond_c A =_{df} \neg \Box A \wedge \Diamond A$, which is equivalent to $\Diamond_c A =_{df} \neg \Box A \wedge \neg \Box \neg A$.⁸ Substitute A with the term logical formula SaP ,

⁷[9, p. 6]. They use a triangle symbol turned upside down for the modal operator "it is contingent that..." As contingency is one subtype of possibility, I use the symbol for possibility with an addition of "c" for contingency.

⁸Aristotle already knows the mutual definition between "possible" and "necessary". "Impossible" is simply the negation of "possible" and does not require its own symbolization.

or with its first-order predicate logic reformulation⁹, we have $\Diamond_c \forall x(Sx \rightarrow Px) = \neg \Box \forall x(Sx \rightarrow Px) \wedge \neg \Box \neg \forall x(Sx \rightarrow Px)$, and by applying the mutual definition of possible and necessity, we have the formula $\Diamond_c \forall x(Sx \rightarrow Px) = \Diamond \neg \forall x(Sx \rightarrow Px) \wedge \Diamond \forall x(Sx \rightarrow Px)$, in difference to Falessi and Schang, the negation of one part of the conjunction is the negation of the proposition which the modality *de dicto* is about, in Ockham's word, the *dictum*, and not the internal relationship between subject term and predicate term inside the *dictum*. Therefore, Falessi and Schang's interpretation of contingency in Ockham is not correct.

A second problem is their interpretation of *de re* modality in general. Falessi and Schang give a syntactical description for *de re* and *de dicto* modalities, but by taking them as equivalent, they do not bother to distinguish between a semantics for *de re* and a semantic for *de dicto* modalities. According to them, in order for the A-type statement to be necessary, it is required that all individuals referred to in this statement have the corresponding property *P* in all possible worlds. In their interpretation, a necessary A-type statement means the same in the *de re* and *de dicto* meaning, i.e. the statement "It is necessary that every *S* is *P*" (*de dicto*) has the same meaning as "Every *S* is necessarily *P*". ([9, pp. 13–14]) There is just one semantics, which is reduced to the satisfying relation between individuals and properties at certain worlds. So, for necessity, this can be reduced to the requirement that all individuals in the Domain have the property *P* in all possible worlds. In their model there is just one single domain of individuals, and for each world, variables are assigned with values from the same domain *D*. Thus, for example, in their minimal model, a proposition in form of SaP is necessary, when the two individuals *a* and *b* of domain *D* are both *S* and also both *P* in both worlds *w*₁ and *w*₂. ([9, p. 13]) This assumption is however not grounded at all. Not only because it does not match what Ockham says, but also because Falessi and Schang assume for their modal system that is meant to interpret Ockham's modal logic that *de re* and *de dicto* modal statements are equivalent, this however presupposes that Ockham's modal logic should be a system which includes Barcan formula and its converse. Ockham does talk about the equivalence of *de re* and *de dicto* modal propositions in which a predicate is said of an individual, as when formalized, they are indifferently to be represented as $\Box Pa$, while *a* is an individual constant, just take necessity for an example. (*SL* II, 10, [12, p. 112]) In this case, if there is a predicate *P*, so that for any individual *x* in the Domain, $\Box Px$ is true, i.e. $\forall x \Box Px$ is true, then $\forall x \Box Px$ is also true. In this sense, Ockham could have implicitly acknowledged the Barcan formula and its converse. However, Falessi and Schang's assumption is more than this. They tacitly assume that a formula

⁹In the research there are discussions regarding which is the adequate representation of Aristotelian universal affirmative propositions, as for Aristotle, truth condition of this type of proposition includes the existence of things the subject term refers to, but as this is not an issue here, we take over the way how Falessi and Schang formalize SaP with first-order predicate logic.

in form of $\Box\forall x(Sx \rightarrow Px)$ implies $\forall x(Sx \rightarrow \Box Px)$. However, Barcan formula BF: $\forall x\Box\varphi \rightarrow \Box\forall x\varphi$ and its converse CBF $\Box\forall x\varphi \rightarrow \forall x\Box\varphi$ only allow the inference from $\Box\forall x(Sx \rightarrow Px)$ to $\forall x\Box(Sx \rightarrow Px)$, and hence $\forall x(\Box Sx \rightarrow \Box Px)$, if we assume that Ockham's modal syntax includes the modal propositional logic theorem *K*. Ockham's modal propositions *de re* have rather the form $\forall x(Sx \rightarrow \Box Px)$, and not $\forall x\Box(Sx \rightarrow Px)$, because in difference to modal propositions *de dicto*, the modal propositions *de re* concerns the internal structure of a categorical statement of the Aristotelian term logic.

This is clear from Ockham's own text: In modal propositions *de re*, the modality expresses the form in which the predicate belongs to the objects to which the subject term refers, (*SL* II, 10, [12, p. 112]) thus, only singular necessary statements are equivalent in the *de re* and *de dicto* sense. Ockham says in his *Summa Logicae* II, 10 that the necessary sentence *de dicto* "That every man is an animal is necessary" is true, but the corresponding *de re* form "Every man is necessarily an animal" is false ([12, pp. 111–112]), from which it is quite obvious that universal affirmative statements on necessity are not equivalent in their *de dicto* and *de re* reading. Other examples can be found, for example, a modal proposition *de re* is true but contingent, so it is not a *de dicto* necessity. In section 5 we shall examine Ockham's text closely, in order to get a more accurate and clearer picture of his distinction between modal propositions *de re* and *de dicto*.

4.3 Problematic philosophical presumptions involved in Falessi and Schang's reconstruction

Falessi and Schang begin with Ockham's own account of modalities, but they do not go far enough to examine Ockham's own explanation of the modal terms, which is crucial for a better understanding of Ockham's modal logic. In their reconstruction of a formal semantic for Ockham's modalities, they take presumptions for granted, without asking whether they do justice to Ockham's own understanding. Beside of the logical problems sketched above, there are also philosophical presumptions in their reconstruction which we shall put under a critical examination.

One presumption is that possible world semantics is adequate for interpreting Ockham. Using relational semantics involving possible worlds for the semantics of medieval modal logic was proposed by Simo Knuuttila, according to whom medieval philosophers, most prominently Duns Scotus, discovered synchronic possibilities that imply the idea of possible worlds. ([19, p. 138]) Falessi and Schang are obviously inspired by this idea as Knuuttila's work is included in their list of reference. This might have been true in case of Duns Scotus and other medieval logicians, but caution is asked for in case of Ockham whose nominalist ontology would be hostile to the idea of introducing entities like synchronic alternatives. As Knuuttila points out, Ockham

does not allow positing possible beings that are distinct from real existent things. ([19, p. 146]) Therefore though a possible worlds semantics is a powerful tool for explication of modal terms, the question of adequacy still has to be answered. If we look carefully at Ockham's own definition of "necessity", a temporal interpretation suggests itself quite naturally. According to Ockham, a proposition is called necessary (*de dicto*), "because it is true if it exists and cannot be false", and "not because it is always true". ([12, p. 111])

Falessi and Schang assume that Ockham's necessity is generally a *S5* necessity. ([9, p. 12]) However, *S5* is first of all a name for a certain modal system that contains a number of certain inferential rules. Accordingly, *S5* necessity is the necessity in *S5* system which represents the idea of universal necessity, that is, necessity in all possible worlds, or displayed in a Kripkean model, as "a universal accessibility relation" or "an accessibility relation that is an equivalence relation". ([37, pp. 53–54]) Therefore, in an *S5* system, the operator $\Box$ behaves in that way so that it is equivalent with *S5* necessity. ([13, p. 105]) But this does not mean that if some propositions are necessary in the *S5* sense, the model containing them must be syntactically *S5* too. This is not the case because *S5* necessity is a special kind of necessity, namely, the tautological or alethic necessity. ([13, p. 104]) In a modal system, we can accept different ways of understanding "necessity", provided that we use different operators to mark the differences. Therefore, there is no need of worrying about whether there can be an *S5* necessity in Ockham's modal logic, if we set up a time-based semantic model for Ockham, as we can use $\Box_5$ for alethic necessity, by interpreting it as a sort of omnitemporal truth with certain restriction due to Ockham's conception of proposition as real articulated linguistic or mental entity. As we shall explain in section 5.

Summing up, we have to find out a more adequate description for Ockham's *de re* and *de dicto* modalities, as Falessi and Schang's account does not quite match what Ockham's own understanding. There is *S5* necessity in Ockham, but this should not exclude other understandings of necessity in Ockham's modal logic, since he talks about the historical necessity in the sense of Diodorus and Aristotle's *De Interpretatione* 9, we have to find a semantic model in which these understandings can be described. Although we agree with Falessi and Schang that a relational semantic is a proper tool for interpreting Ockham's modalities, we do not need to postulate possible worlds. Given Ockham's own nominalist ontology, it seems to be more appropriate to use time-points as contexts. We will take up these ideas and sketch out a new approach to semantics for Ockham's modalities in section 5.

5 A New Approach to Semantics for Modalities in Ockham

5.1 Philosophical preliminaries

As Ockham gives a clear definition of necessity in the composite sense, it provides a good start-point for the search for an adequate semantics for Ockham's modalities. A modal sentence with necessity in the composite sense is true, if and only if the dictum contained in this sentence is true, when it is (formulated) and cannot be false. (*SL* II, 9; *OP* I, 275) With this formulation of truth-condition for a necessary proposition in the composite sense i.e. in the *de dicto* reading, Ockham seems to depart from the omnitemporal definition of necessity which we can find in Aristotle. But his reason is not to dispense with the Aristotelian tradition, but to stress the special ontological status of propositions. As for medieval logicians in general, a *propositio* is not an abstract entity as the modern proposition is in the Fregean sense, but is something more like a token sentence. ([6, p. 116]) For Ockham, it is sufficient for a sentence to be a truth-bearer, when it is mentioned. ([1, p. 37]) But if we decide not to follow Ockham in this ontological presumption, which is not crucial for his logic, nothing prevents us from saying that a necessary proposition *de dicto* is a proposition that is always true. But Ockham is not talking about a statistical interpretation of necessity, because what he wants to express is that there are true propositions that are independently true of the context in which they are articulated. According to Hintikka, Aristotle interprets modalities by referring to token reflexive indefinite sentences, therefore, universal sentences like "every human being is rational" is true and necessary, as it is always true. ([16, pp. 63–70]) But for Ockham, universal propositions like this are not necessary but contingent, because things the subject term supposits for are not always existent. So according to Aristotelian logic, in difference to the modern predicate logic that treats universal quantification in form of a conditional, the existence of the things the subject term stands for belongs to the truth-condition of a universal affirmative sentence. Instead, for them to be necessary, they need to be reformulated into propositions like "every human being is able to laugh", or into hypothetical propositions like "if there are human beings, they are rational". (*SL* I, 22, 24, 26; *OP* I, 72, 75, 80, 87) By doing so, Ockham is turning a context dependent sentence into a sentence whose truth is not dependent on the time point of its utterance. As the truth of a sentence like "every human being is rational" is dependent of the time points when there are human beings. It is true that recognising these sentences to be true presupposes an essentialist ontology, to which Ockham is very well committed to, as is clear from his *Commentary on Porphyry's Isagoge*.¹⁰ Ockham's view matches quite well with the so-called Kripke-Putnam-Essentialism,

¹⁰Ockham writes there that if Socrates does not exist, then he is neither rational nor a human being, but if he exists, he is necessarily a human being and rational. See *In Isagoge, cap.* 11, *OP* II, 117.

which says that if in every possible world, every sample of the natural kind K possess the property p , then p is an essential property of K , ([39, p. 159]) which can be expressed in the following formula: $\Box\forall x(Sx \rightarrow Px)$, while S is a term denoting a natural kind, and P denotes the essential property p . This shows, that essentialism is expressible through *de dicto* necessity, and not through *de re* necessity. Sentences that express natural and metaphysical necessity can be transformed into the forms of conditionals, and they are *de dicto* necessary which is true in all possible worlds, and in Ockham's case, without having to posit possible worlds, we can use time points as context, therefore, they are true at every time point (when articulated). Ockham does not give many examples of necessary propositions *de dicto* that express alethic truth but they fulfil the same truth-condition as being true, whenever they are articulated and cannot be false. We shall use the usual symbol $\Box$ for this kind of necessity.

But while looking closely to Ockham's definition of the above discussed necessity, the additional clause that it cannot be false is remarkable. Of course, it is possible to take it as a parallel remark that says the mutual definition of the modality necessary and possible. But it can be understood in the way that description of necessity relies on the notion of possibility. In deed some modern interpreters regard Ockham's notion of modalities as primarily logical. Lili Alanen finds that possibility in the absolute sense for Ockham is a predicate of sentences and as a pure logical modality it doesn't require any cause. ([2, p. 175]) This implies that we do not need to resort to God's power for the interpretation of possibility in Ockham.

But how we should interpret Ockham's possibility proves to be difficult, given that we take Ockham's own remark verbally and interpret the *de dicto* necessity as being true all the time (or whenever articulated). It seems that if we give a temporal interpretation to Ockham's modalities, especially in regard of the modality possible, there is no escape from what Hintikka call's the Principle of Plenitude, that is, everything that is possible must be true at least once, which implies that any possibility would have to be realised at one time point, therefore, in case of Aristotle, possibility should "be realized at some particular moment of time". ([17, pp. 102, 105]) This might be the reason that modern logicians tend to rely on possible worlds model than on a time based model for modalities. However, a way out of this dilemma is that while accepting the Diodorean interpretation of necessity as "always true", we do not need to interpret possibility as "sometimes true", Broadie suggests that we understand a modal sentence on possibility not as "sometimes true", but as "its existence does not imply its falsity". ([5, p. 59]) This seems to give a reasonable account of logical possibility for Ockham, as this paper is not a metaphysical investigation about the grounds of modalities in Ockham, it is for this moment sufficient to take this interpretation of logical possibility as primary, while necessity, while being mutually definable with possibility, can be semantically grounded on possibility.

However, there are another type of modalities in Ockham which are more difficult, namely modalities conjunct with tense. What is in question here is not what Rescher and Urquart call temporal modalities, which they mean modalities with tensed interpretation ([33, p. 125]), but modalities concerning propositions in certain tense. Future contingency expresses the philosophical intuition of choice of action open to us. For his Ockham interpretation, Arthor Prior combines in a sense the possible worlds model with a temporal model, therefore set up a branching time model which he dubbed the Ockham tree, which is used for the illumination of the so-called future contingency. ([15, p. 3411]) However, this model introduce two possible alternative histories which means, philosophically, that we have to postulate alternative possible entities for the future, which is not in line with Ockham's nominalist ontology. Ockham grounds the two sided-possibility, which is also called contingency, on the capacity for opposites. Dun Scotus talks also about this kind of capacity and introduces the notion of synchronic instances of nature, among them, one is actualised while the other remains non-evident. Ockham however rejects this idea, for him, this capacity, which is not actualized, is not a real capacity and should not be posited. The free will of a rational being is a typical example. ([41, pp. 71–72]) For Ockham, this kind of contingency is based on something real, namely, the power of acting of rational agents. This modality is to be distinguished from logical possibility, and not to be defined simply as $\Diamond p \wedge \Diamond \neg p$, as time aspect must be integrated. Ockham explains in his *Treatise on Divine Predestination and Future Contingency* that the contingency of will that acts as a free cause can be understood correctly in two ways, in one way, the will, which is a real power and thus exists also prior to its act at t_1 , for example, causes this act at t_1 contingently. Logically, this can be described as the proposition “the will causes its act at t_1 ” is true, while at a time point after t_1 , it is true that the (same) will does not cause the same act it caused at t_1 . ([41, p. 76]) If we apply possible worlds semantics for the interpretation of the contingency of human acts, we can interpret an act which is expressed by the proposition p that it is true in w_i , while its opposite $\neg p$ is true in w_j , while $i \neq j$, this can be interpreted also as related to time-points, so that a proposition p is contingent, if p true at t_i , while its opposite $\neg p$ is true at t_j , while $i \neq j$. We can also apply Prior's tense operator to express this idea: p is contingent, if $p \wedge F\neg p$.

What is important is to understand a well-formed proposition in future tense that expresses free acts of rational agents, a proposition in form of Fp is contingent, but if p , which is a present tensed proposition expressing such an act, is true at t_j , then for every time point after t_j , $\mathbf{P}(p \text{ at } t_j)$ is true, as Ockham writes that:

Every proposition that is merely about the present, if it is true, has a necessary proposition about the past. But by the hypothesis “the will wills X at t_1 ” will be necessary forever after. Therefore after t_1 , “the

will did not will X at t_1 ” cannot be true. ([41, p. 73])

A well-formed future tensed proposition is contingent, because it is undefined in time, as long as the same will exists, any time in the future can provide a context in which an alternative act of will can happen. What Ockham means by mentioning a time point after t_i is not to point out to a definite time point at which $\neg p$ will be true. Given the density of the time structure, seen from the perspective of t_i to time points after it, there is an unlimited number of time points for $\neg p$ to be true. Therefore, using the tense operator “**F**” is better than quantifying over time points which will have unwelcome consequence that we have to point out to a certain time at that $\neg p$ is true, and thereby slip back to the Principle of Plenitude. The idea that the future is open can be captured by the undefined time of a future event. We can only specify its time point of happening, when it happens now, and this makes it necessary afterwards, as for its logical expression, we can add time to it to make it a complete sentence in the Fregean sense and not token reflexive any more. Modalities concerning tensed propositions are therefore different to alethic modal, therefore, we use the symbol $\Diamond_{fc}$ for future contingency, and $\Box p$ for the necessity of the past.

5.2 Informal account of a semantics for Ockham’s modalities with formal description of several central features of Ockham’s modalities

For a new approach to Ockham’s semantic on base of time, we introduce a quadruple $\langle \mathbf{T}, \mathbf{D}, \mathbf{R}, \mathbf{a} \rangle$, with “**D**” as the domain of quantification, “**T**” as the set of time points, while “**R**” is the set of binary relation on **T** representing the temporal order, so that if $i < j$, then $t_i R t_j$, but not $t_j R t_i$ (i and j are real numbers, as we map sequences of time points to the sequences of real numbers. As time is dense so that there is always a number that is larger than i but smaller than j). With this structure, we try to capture the irreflexivity and density of time. However, we have to introduce time related domains, and objects in each time-related $\mathbf{Dt}$ have an identity relation to objects belonging to other time related domains, for example, if the range of the existence of object a is from t_i to t_j , then a belongs to domains ranging from $\mathbf{Dt}_i$ to $\mathbf{Dt}_j$, while eternal things belong to every time related domain.

5.2.1 Alethic modalities

Ockham gives an account of alethic modalities in the *Exp. in libr. Perih* and the *SL*, which are of importance for conversions of modal propositions and inferential relationships between modal propositions. Ockham makes clear that there are four different alethic modalities. Both the modal expressions “possible” and “contingent” can be understood in two different ways: They can mean “not impossible”, in this meaning, they include what is necessary, or they can mean “neither necessary

nor impossible”, which excludes necessity. (*Exp. in libr. Perih.* II, 5, *OP* II, 466) However, the expression *contingens* is more frequently used for the two-sided possibility, which is also called *contingentia ad utrumlibet* by Ockham. (*SL* II, *cap.* 27, *OP* I, 334) Therefore, when we talk here about contingency, we mean the two-sided possibility.

Necessary and possible are mutually definable, with “possible” as the more fundamental one, as it is used to define “necessary”. The meaning of “necessary” can be exhibited by the truth condition for a modal sentence with this modality. For Ockham, the *dictum* contained in a modal sentence on necessity doesn't need to be always truth, it needs only to be true, when it is formulated and cannot be false. Likewise, a modal sentence on impossibility is true, whenever it is formulated, and cannot be true. (*SL* II, 9, *OP* I, p. 275 line 72–79)

As already explained above, while it is granted that Ockham does have notions of metaphysical and natural modalities when dealing with natural philosophy and theology, within the framework of his logic, they can always be adequately expressed by sentences with logical modalities. Contingency, understood as the two-sided possibility, is according to Ockham equivalent to “possible and not necessary” (*SL* III-3,13, *SL* I, 643), given the definition of “necessary” as “not possible not”, we obtain the definition of “contingent” as “possible and possible not”, that is to say, sentence *A* is contingent iff *A* is possible and non-*A* is also possible.

It is easy to capture the alethic necessity with the time-based model introduced above:

$$\mathbf{a}_t(\Box A) = \top \text{ iff for each } \mathbf{i}, \mathbf{a}_i(A) = \top$$

Given the mutual definition of “possible” and “necessary”, we have the following definition for “possible”:

$$\mathbf{a}_t(\Diamond A) = \top \text{ iff for some } \mathbf{i}, \mathbf{a}_i(A) = \top$$

And accordingly, the definition for “contingent” in the sense of two-sided possibility:

$$\mathbf{a}_t(\Diamond_c A) = \top \text{ iff for some } \mathbf{i} \mathbf{a}_i(A) = \top, \text{ and for some } \mathbf{j} \mathbf{a}_j(A) = \perp,$$

As we have seen, alethic modalities are used to express the nature of things and in this sense they are universal propositions, they do not concern events or actions in a person's life history or states of an individual thing, therefore we shall not be bothered with the metaphysical problem whether possibility should be realized at one time point here, and talking about time points should not entail the very existence of certain time points because the evaluation is done from one time point and we have access

to knowledge of states of things at earlier time points and by natural laws we can predicate the states of things at later time points. But in case of contingent acts, there is no way to evaluate it beforehand, as Boethius makes it also clear in his explanation of Aristotle's Sea Battle Problem from *De Interpretatione* 9. According to Boethius, propositions about future contingent events are all indefinite in their truth-value. (*OP* I, 710–711; *OP* II, 516 1.229–238; [22, p. 101]) Ockham follows Boethius in this regard, as he regards propositions on future contingents as indeterminate regarding their truth-value. ([35, p. 289])

5.2.2 *De re* and *de dicto*

Based on the implication relations among different modalities, Ockham outlines a detailed syntax for modal proposition in the *de dicto* sense, which is worked out in details by Wolfgang Lenzen. Modalities *de dicto* are used to express relationship between terms of natural kinds and their essential properties. Modalities *de dicto* are not equivalent with modalities *de re* though, because for Ockham, the *de re* sense is related to individuals and not to kinds. This is clear from the description given by Ockham in his *Summa Logicae*:

[For the truth of propositions in the *de re* sense], it is required that the mode expressed in such a proposition be truly predicated of a non-modal proposition in which the very same predicate is predicated of a pronoun referring to that for which the subject-supposits. ([12, p. 112])

Therefore, if the subject term is a pronoun or a proper name picking out a thing directly, modal propositions are equivalent in the *de re* and *de dicto* sense. Let's take object *a* that is predicated by *P* necessarily in the *de re* sense, then $\Box Pa$, given Ockham's omnitemporal interpretation of necessity *de dicto*, then *Pa* is always true, as we can see, only eternal objects are to be said necessarily in the *de re* sense, while for things with temporally limited existence, only necessary propositions in the *de dicto* sense can be formed. But it is a different case when the subject term is a general term. Therefore, Ockham gives the example "A being which creates is possibly not God" which is in the *de re* sense impossible, but not in the *de dicto* sense. Because the term "a being which creates" supposit for God, but the proposition "God is not God" involves a contradiction and is therefore not possible. But the proposition "A being which creates is not God" is possible, because "create" and "God" are only related to each other contingently through the free will of God to create. (*SL* II, 10, [12, p. 112]) A analogous example is given for necessity in the same place. Therefore, we cannot say the formulas $\forall x(Sx \rightarrow \Box Px)$ and $\Box \forall x(Sx \rightarrow Px)$ are equivalent to each other.

To capture Ockham's understanding of *de re* modalities in propositions that contains general terms in their subject terms, we should evaluate them from perspective

of the time of their articulation. When at time point t_i such a proposition SaP is articulated, the quantifier range over existent things at time point t_i , that is, the domain $\mathbf{D}t_i$, and the subject term S picks out things that satisfy Sx at time point t_i , so if all these things are P , or satisfy Px , and for all these thing, also Px is true, while we apply the definition given above for alethic necessity, so that for everything that is S at time-point t_i , $\Box Px$ is true means that these things exist eternally, otherwise Px cannot be true for each time-point t , and that they satisfy Px at each time-point t . However, though these things are picked up by term S at time-point t_i , and they exist at each time point, but they do need to be S at each time point. In the proposition Ockham uses as an example, that is, “A being which creates is necessarily God”, the term “a being that creates” picks out at time point t after the creation¹¹ the eternal existent God, and let a stand for this being, and a belongs to all time related domains, so that at each time point t , the proposition “God is God” is true. We can say that for Ockham, the Barcan formula and its converse do hold, as far as the formula is concerned, but there is no equivalence between formulas like $\forall x(Sx \rightarrow \Box Px)$ and $\Box \forall x(Sx \rightarrow Px)$.

5.2.3 Temporal modalities

Ockham understand under “future contingents” propositions about future event that does not belong to the realm of natural necessitation or logical necessity, but to the realm of freedom of rational agents, these propositions are contingent in the sense of temporal contingency, and not in the sense of logical contingency, thus, future contingency is symbolized by $\Diamond_{fc}$, while the necessity of the past is symbolized as $\Box p$. They can be both defined within the framework $\langle \mathbf{T}, \mathbf{D}, \mathbf{R}, \mathbf{a} \rangle$ introduced above, and we add tense operator $\mathbf{F}$ for “it will be the case, that ...” and $\mathbf{P}$ for “It was the case, that ...”¹²: “ A at t_i ” is a proposition in present tense which says that an event described by A (which is also in present tense) happens at t_i .

$$\mathbf{a}_j[\Box p(A \text{ at } t_i)] = \top \text{ iff for each } \mathbf{j}, \text{ such that } \mathbf{t}_i \mathbf{R} \mathbf{t}_j, \mathbf{a}_j[\mathbf{P}(A \text{ at } t_i)] = \top$$

It is clear that the necessity of the past does involves the omnitemporal truth of $\mathbf{P}(A \text{ at } t_i)$, because before time point t_i , $\mathbf{P}(A \text{ at } t_i)$ is not a well-formed formula, so for time points before t_i , it does not make any sense to assign $\mathbf{P}(A \text{ at } t_i)$ any truth value. And true past-tensed propositions are only necessary in the temporal sense at certain time points, namely, only at time points after the time of happening for A . In

¹¹If we assume with medieval theologians that before creation there was nothing that created, and that after creation of the world there is a so-called continuous creation so that we can henceforth refer to God as “a being that creates”.

¹²As P is used by Falessi and Schang as symbol for property, and in term logic P is the traditional symbol for predicate, we use here the bold character $\mathbf{P}$ and $\mathbf{F}$ as temporal operators

this the necessity of the past differs from the alethic necessity which is necessary at all time points.

And for the contingency of the present, for example the contingency of the present-tensed proposition “I am typing” which is true at time point t_i .

$$\mathbf{a}_i[\Diamond_{fc}(A \text{ at } t_i)] = \top \text{ iff for some } \mathbf{j}, \text{ such that } \mathbf{t}_i \mathbf{R} \mathbf{t}_j \mathbf{a}_j(A) = \perp$$

We should note that the talk about “some” here should not imply or introduce any real existence.¹³

For contingency of the future then:

$$\begin{aligned} \mathbf{a}_i[\Diamond_{fc}\mathbf{F}(A \text{ at } t_i)] &= \top \text{ iff for some } \mathbf{j}, \text{ such that } \mathbf{t}_i \mathbf{R} \mathbf{t}_j \mathbf{a}_j(A) \\ \text{not definitively} &= \perp \text{ or } \top, \text{ and } \mathbf{a}_j[\mathbf{F}(A \text{ at } t_i)] \text{ not definitively} = \perp \text{ or } \perp \end{aligned}$$

In regard of future events that do not happen out of necessity, for example predictable physical events governed by natural laws, the corresponding future-tensed propositions are contingent, because, looking from time points before the time of happening the future-tensed proposition refers to, there is no way to assign a definite truth-value to them at these times points.¹⁴ Therefore, these future contingent propositions are contingent at these time points. Ockham uses Aristotle’s example of the sea battle tomorrow, and according to him, there is no way for any one of the contradictory pair of propositions that there will be a sea battle tomorrow and that there will not be a sea battle tomorrow to be in the definite way true or false. Before the future event the proposition refers to actually happens, the same future-tensed proposition is contingent, but not contingent afterwards.

6 Conclusion

So far, we have attempted to outline a time-based semantic for Ockham’s modalities. We have identified at least two types of modalities that should be defined differently: Alethic modalities and temporal modalities, however, both can be defined within the same time-based semantics. We have shown that propositions expressing natural or metaphysical modalities can be reformulated as conditionals making them necessary or possible *de dicto*, so they can be expressed by alethic modalities. Modalities *de re* and *de dicto* are not equivalent and we have tried to give an informal description of how *de re* modalities behave, by relating domains to time-points. The

¹³Free logic might be a good alternative to look at problems of denotation and quantification.

¹⁴Some logicians distinguish between “true” and “definitely true” so there is a possibility to acknowledge both the claim that future contingent propositions are true and that they are not definitely true. But here is no room to elaborate on this topic.

main advantage of this time-based semantics is that it allows us to make Ockham's temporal modalities explicit and clear. As the aim of this paper has never been to provide a complete formal semantics for Ockham's understanding of modalities and modal logic, and since medieval logicians work with natural language and intuitions; rather than constructing logic in the axiomatic way familiar to modern logicians, it would not be an adequate representation of Ockham's understanding to attempt fully formalised reconstruction. The problem of adequacy when using modern logical tools is already pointed out by Stephen Read and Catarina Dutilh Novaes, as mentioned earlier in this paper. However, Novaes's idea that we can provide formal descriptions of features that are already present in historical theories is helpful. Following this lead, we applied modern logic to capture several features of Ockham's theory of modalities, which has proved to be a useful work as it gives us a clearer picture of Ockham's modalities, especially his temporal modalities.

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解释奥卡姆逻辑中模态概念的新方案 ——建基在时间上的语义学

江璐

摘 要

二十世纪七十年代所做的勾勒奥卡姆模态的诸尝试仍然有意义，因为它们指出了奥卡姆模态的时态特性。然而，这个重要的洞见在之后的几十年中被忽视了。法勒西与尚格（2023）是最近少数致力于详细刻画奥卡姆模态的尝试之一，此文使用可能世界语义学，却忽视了时态的一面。本文先对现代的奥卡姆模态语义学进路进行一个总体勾勒，随之讨论关于如何呈现中世纪逻辑的方法论问题，并对奥卡姆的模态概念阐述。在此之后，本文细致考虑了法勒西与尚格的进路，并且对照奥卡姆的原文与奥卡姆自己对模态的理解对其加以评价。这样，我们得以确立法勒西与尚格的重构中哪些地方对奥卡姆的模态概念理解之呈现为不恰当的，从而推动我们回到奥卡姆的原文，并借助现代的逻辑工具来呈现他关于模态概念的理解。最终，我们得出一个不需要分支时间的建基在时间上的关系语义，它能够呈现奥卡姆模态概念的几个关键特性。