

Late Medieval Views on Formal Consequence

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Abstract. The paper presents by means of examples various medieval classifications of consequences (*consequentiae*, or inferences) into formal and material or natural and accidental. Validity of a consequence was in the middle ages understood through the modal criterion that the antecedent cannot be true without the consequent. Medieval logicians employed two main intuitive ideas for separating a privileged class of valid consequences: (1) conceptual containment (the understanding of consequent is included in the understanding of the antecedent), and (2) logical form defined through the substitution principle (the consequence is valid for all terms). The concept “formal” (*formalis*, or *de forma*) became the standard term for the privileged class. Similarly “material” (*materialis*, or *de materia*) was adopted for the inferior class of non-formal inferences. The substitution principle carefully formulated and perhaps invented by John Buridan may seem particularly appealing to many modern logicians. In the fourteenth century it did not gain universal acceptance. When accepted, it was considered as secondary to the principle of conceptual containment, or in modern terms analytic validity.

1 Validity and Consequence Theory

Validity is a property many inferences have, but some inferences possess special, even logically privileged validity. Medieval logicians called them “formal consequences” or “natural consequences”. While we can trust that readers familiar with basic contemporary logic have some idea of what consequence in general is, the words “formal” and “natural” can in this context invoke misleading associations. We use the terms in accordance with medieval practice, and thus they have nothing to do with formal calculi, natural deduction, or natural language. In the course of this paper, we hope to shed some light on the content of the medieval usage. Indeed, our main aim is to show what kind of validity a formal consequence was taken to have in various medieval logical theories.

In the first place, we consider a few late medieval classifications of consequences that presented some of them as *natural* or *formal* in contrast to others. We choose to represent consequences as inferences, though obviously there is a regular correspondence between inferential *consequentiae* (p , therefore q) and conditional propositions

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(if p then q).¹ And further, let us decide that an inference is called a consequence only if it is correct, a “good consequence”, as the medieval logicians used to say.

Let us start with a few relevant examples that will be used throughout the paper. We shall try to apply the medieval classification to these cases of inference. So, let us consider the following five instances (based on medieval examples):

- | | |
|---|---|
| <p>(A) Every cat is an animal
 <u>Some cat is black</u>
 Some animal is black</p> | <p>(B) <u>A man is running</u>
 An animal is running</p> |
| <p>(C) Socrates is sitting
 Socrates is not running</p> | <p>(D) <u>A man is a donkey</u>
 A stick stands in the corner</p> |
| <p>(E) <u>Socrates is sitting and Socrates is not sitting</u>
 A stick stands in the corner</p> | |

It seems that all of these inferences are valid in some sense of the word “valid”. However, their validity seems rather different in each of the cases. Today we are accustomed to call (A) formally valid, and accept it without scruples. In (B) and (C) logicians might disagree about the formality or logicity of the consequence, but presumably few would completely refuse its validity. In contrast to these, (D) and especially (E) look somehow paradoxical and have caused a lot of discussion among logicians.

Our main interest is in the types (B) and (C), and the reason is that those cases serve best for distinguishing between various ways in which the strictest kind of validity of a consequence could be understood. Many, but by far not all, of the logicians we review used to think that inferences like (B) and (C) were “natural” and had the strictest kind of validity. Connected to naturalness, we find the problem of the correct criteria of validity. Discussion of this problem seems in the early fourteenth century to yield the notion of formal validity based on the substitutivity of the non-logical terms. For a present-day reader, such an idea can be especially attractive as a definition of strong validity. And from this viewpoint, the validity of (B) and (C) may indeed seem weaker than formal, because they do not hold with arbitrary substitutions. For a present-day reader, they may even be seen as somehow dependent on analytic truths. Despite the merits of the account based on arbitrary substitutions, medieval logicians often preferred other ways of explaining the validity of logical inferences, perhaps exactly because of deciding not to see any difference in the degree of validity between (A) and (B) or (C). This forces us to ask for the logical relation between the

¹Concerning the decision in this question see [19], where it is shown that late medieval logicians were aware of the distinction between the two and mainly favoured the interpretation of consequence as inference.

antecedent and the consequent of inferences like (B) and (C).

As a background, it is perhaps best to say first a few words about how we understand the history of medieval *consequence theory* in general. Popular accounts still often give the impression that medieval logic was mostly concerned with syllogisms. But a quick look at relevant texts shows that such an idea is completely mistaken: They go through a large number of logical and semantic questions in a fairly established sequence of sections, and there is more about terms and propositions than about different inference modes. The growth of logic in late thirteenth and early fourteenth century brought a number of novelties, and one of them was that philosophers incorporated a section about consequence theory into their logic books. Fourteenth-century logicians then examined the doctrine of inference in general under the title “*consequentia*”. Consequence theory became a common chapter in logic texts, and separate treatises *De consequentiis* started to appear as well.²

The historical origin of the theory of consequences is somewhat controversial. It has probably some connection to the old tradition of *topics*, based on Aristotle and Boethius, where people had long been studying possible dialectical argument types. Similar investigation of inferences was later often continued in the theory of consequences, but it is by no means clear that this theory can be seen as a simple expansion or elaboration of topics.³ There appear to have been other important sources, too, at least the doctrine about the so-called hypothetical syllogisms and the thirteenth-century treatises about syncategorematic words “if” and “unless” among others. The best known treatise in that genre was written by Peter of Spain (edited in [26]). But, undoubtedly, many practices of topics were adapted to consequences; for instance, rules of consequence were called *loci*, like the topical rules of good argument. And it can hardly be a coincidence that, as consequences developed, topics gradually lost importance (until a revival in Renaissance rhetoric).

Anyway, a technical consequence literature emerged around 1300, probably first in England but soon expanding throughout the Continent. As the philosophically most original names of consequence theory we might suggest three prominent logicians who wrote about it between 1320 and 1340: Walter Burley, William Ockham, and Jean Buridan. Their close followers continued an interesting discussion about these questions. After that period, consequence theory was constantly investigated but, oversimplifying, we can say that the emphasis turned from theoretical innovations toward systematic exposition. We meet such a consequence theory in the comprehensive logic of Paul of Venice, and then in the leading fifteenth-century logicians. But in late fifteenth century many authors gave up consequence theory, though others cultivated this traditional field long after 1500.⁴ Then they were mainly interested in

²Some outlines on the historical development of consequence theory can be found in [2, 7, 15, 19].

³Concerning the general development of the medieval topics, see [17, 35].

⁴For an overview, see [5], chapter III.1.

complex special cases and counterexamples.

There are two sides in consequence theory. In the first place, it presents and derives various patterns of correct inference. Later, this task naturally survived, and some authors went into considerable detail when they derived dozens of inference rules, with additional complications for modalities, and so on. But on the other hand, there is the more philosophical issue of what a deduction is and what exactly makes it valid. This inquiry started in the age of Burley, Ockham, and Buridan. Our problem is from that field.

2 Walter Burley

Walter Burley (1274/5 – 1344) wrote his brief manual *De consequentiis* already in 1302, but a systematic general *theory* about the relevant grounds and divisions of consequences was given in the two versions of his work *De puritate artis logicae* (written in the 1320s, edited in [11]).⁵ He places general consequence theory clearly before all special kinds of inference. (That is why he even lays down that propositional logic must in a certain sense precede syllogistics and predicate logic—an unusual observation among medieval logicians.)

Burley's *definition* for the validity of an inference is that a consequence holds if its antecedent *A cannot be true* unless the consequent *B* is also true. A necessary and sufficient condition for this is that the negation of *B* is “repugnant” to *A*. In other words, $A \wedge \neg B$ can't be true; necessarily, if *A* then *B*. For him, this needs no further explanation.⁶ Thus the validity of a consequence is defined with a *modal* criterion, which is similar to modern truth conditions of strict implication or entailment. Numerous medieval logicians seem to have used the same defining criterion, but it was not always taken to be as self-explanatory as Burley takes it.⁷

Burley's first classification among valid consequences divides them into simple ones, which hold always, and those valid only for now (*ut nunc*).⁸ Simple consequences are then divided into *natural* and *accidental* ones. Here we can compare Burley's division with some thirteenth century discussions. For instance, commenting upon Aristotle's *Analytics* in the 1240s Robert Kilwardby implied that some con-

⁵We discuss Burley before Ockham although *De puritate artis logicae* seems to react to Ockham's theory (see [4, 15]). To us, it seems that the main lines of Burley's theory were quite developed already before Ockham wrote his *Summa logicae*.

⁶“The antecedent can never be true unless the consequent is true”;([11], p. 61, *Tractatus longior*) “the antecedent cannot be true without the consequent”. (p. 199 *Tractatus brevior*)

⁷See [3], for instance, the discussion of Albert of Saxony, *Perutilis logica* IV, 1.

⁸Consequence *ut nunc* is one example of the notable role of temporal considerations in medieval logic; for some of its later developments see [5], pp. 130–133. Modern scholars have often been interested in consequences *ut nunc* because they have sought for indications of present-day material implication. We cannot dwell on this subject.

sequences are “natural” or “essential”, “when the consequent is naturally understood in the antecedent”. On the other hand, the logical principle that “the necessary follows from anything” is to be classified as an “accidental” consequence.⁹ In the same spirit, Burley now says that a consequence is natural when the antecedent *includes* the consequent, otherwise it is accidental. A natural consequence holds *per locum intrinsecum*, by an intrinsic connection between the propositions, and an accidental consequence holds because of an *extrinsic rule*.¹⁰

Burley does not explicitly describe the required kind of intrinsic connection, but we may presume that among the five examples given at beginning of this paper, he accepts (A), (B) and (C) all as natural consequences. The presupposed intrinsic rule, apparently, spells out the “inclusion” in the natural consequences. On the other hand, Burley’s example of an accidental consequence is like (D), and he seems to make no difference between cases (D) and (E). He mentions two extrinsic rules for accidental consequences: These two rules tell that anything follows from the impossible, and that the necessary follows from anything. They are the famous “*paradoxes of implication*”. Burley does not directly say that they are the *only* types of accidental consequence, but it is hard to see what others there could be. Already Abelard had pointed out that these consequences lead to problems (*inconvenientia*), suggesting even that the paradoxical results could be excluded by requiring that the antecedent always somehow brings about from itself (*ex se exigit*) the consequent.¹¹ As he continues in a good nominalist fashion, this relation must somehow be sought after in the words and their uses rather than in the things themselves.¹² Burley’s definition seems to reflect the same idea: The paradoxical results of our basic examples (D) and (E) are just accidental consequences. But he insists that, nevertheless, they are correct consequences, though not natural.

Burley’s discussion in *De puritate logicae* takes a stance in what has been identified as a central line of division developing in the second quarter of the fourteenth century. This is the distinction between “English” and “Parisian” conception of what

⁹“Ad primum dicendum quod duplex est consequentia, scilicet essentialis vel naturalis (sicut quando consequens naturaliter intelligitur in antecedente) et consequentia accidentalis.” ([18], p. 1140; translation p. 1141)

¹⁰“There are two kinds of simple consequences. Some are natural, and that is when *the antecedent includes the consequent*; and such a consequence holds by an intrinsic *locus*. Accidental consequence is that which holds by an extrinsic *locus*, and that is when the antecedent does not include the consequent but it holds by some extrinsic rule, like ‘If a man is a donkey, you are sitting’; this consequence is valid and holds by this rule: Anything follows from the impossible.” ([11], p. 61; *Tractatus longior* II, i, 1)

¹¹“Two necessities of consecution can be seen. One which is wider, namely when that which is called the antecedent cannot be [true] without that which is called the consequent. And another which is stricter, namely when not only the antecedent cannot be true without the consequent, but also the former brings about the latter from itself.” ([1], pp. 283–284)

¹²“In asserting consequences one must greatly attend to the properties of the words and their right use (imposition), even more than consider the nature of things.” ([1], p. 285)

“formal” means in the context of logical validity.¹³ As we read Burley’s position, the concept “formal” has at the time of *De puritate logicae* been introduced to *explain* the validity of consequences. His insight is that logically salient consequences hold because of *formal* relations, of one kind or another; and therefore the purest validity of consequences can be called “formal”. Burley was not the first to use the word “formal” in this kind of meaning.¹⁴ Also, the most crucial text in *De puritate logicae* appears to be a reaction to theorizing found also in Ockham’s *Summa logicae*, and after the 1320’s the division into *formal* and *material* consequences is standard in logical treatises.¹⁵ Although Burley’s treatise gives “natural” and “accidental” as the main division among consequences, he appears to use the concept “formal” in trying to explain validity of the logically privileged kind of consequences.¹⁶

Burley takes the position that a formal consequence holds *either* by reason of the form of the whole sentence complex (i.e. because of logical theorems, like our syllogistic example (A)) *or* formally by reason of the terms (i.e. because of their “formal”, conceptual meaning-relations, like the analytic example (B)). For its part, a material consequence holds by reason of the terms but only materially. Burley’s claim that there are two ways for a consequence to be formal. He does not use the principle of substitution, but nevertheless formality by the sentence complex appears to lean into that direction, or to the Parisian conception of formal validity. Correspondingly, formality by the terms appears to lean towards the English conception.

One interesting result is that Burley seems to imply that even inferring anything from a clear contradiction, or example (E), would be formal. Validity of such inferences is a matter of complexes and not of terms. On the other hand, inference from an analytic impossibility, or example (D), appears unclear. We are tempted to think that Burley would see it as material, since “a man is a donkey” is impossible merely analytically as the incompatibility of “man” and “donkey” is not taken to derive from

¹³Cf. [29] and references there.

¹⁴Before Burley a notable case is Kilwardby, who in his commentary of Aristotele’s *Prior Analytics* uses the word to refer to the figure and the mood of a syllogism with the idea that the terms and the propositions are the matter. Kilwardby does, however, also use the word to explain the ground for or type of validity at issue. Thus, an inference may be valid “*quantum ad formam*” or “*gratia formae*”. However, his discussion remains far from being systematic in this respect. (See eg. [18], 1094; translation 1095. For discussion, see [40].)

¹⁵[21] contains an interesting discussion of this change.

¹⁶“It must be said that formal consequences are of two [kinds]: One, where it holds because of the form of the whole complex; and such consequences [include] conversion, syllogism, and thus also other consequences, which hold because of the whole complex. And another is formal consequence that holds because of forms of the simple [parts], as the consequence affirming from the inferior to the superior is formal, and nevertheless it holds because of the terms. Thus a consequence can hold because of the terms in two [ways], for it either holds materially because of the terms, or it holds formally because of the term, that is because of the formal [nature] of the terms.” ([11], p. 86; *Tractatus longior II*, i, 3) Burley does not define his terms, and he uses them only in answering to possible objections.

a formal relation between the terms. On this reading, (C) would be similarly material, unless “sitting” and “running” are taken to be “formal” contraries rather than merely analytic.

We think that Burley did not learn to apply the word “formal” to validity from Ockham, because he uses the word already in his *De obligationibus*, taken to be written 1302. In a certain subspecies of the so-called obligational disputations of the 13th century, the respondent undertook to defend an impossible sentence as a truth (*positio impossibilis*). In his treatise, Burley discusses a text questionably attributed to William of Sherwood; to avoid judgement, let us call the author Master W.¹⁷ There, one *positum* to be defended is “a man is not an animal”. But because men are animals, it would seem that it immediately follows by a natural consequence that “an animal is not an animal”, which is a downright contradiction and inadmissible. The core inference at issue is as follows:

- (I) (1) A man is not an animal
 (2) An animal is not an animal

Both Master W and Burley agree that (1) can indeed be admitted and defended in an obligational disputation, and both agree that normally (2) does follow from it by a natural consequence (similar to our basic example (B)). Master W solves the problem by blocking the consequence with a conceptual change: He says that when (1) is posited it is also posited that the concept “animal” is no more included in the concept “man”.¹⁸

Burley makes no use of any such change of concepts. Instead, he restricts the class of natural consequences to be used in the disputation. He advises the respondent to accept (1) but to deny (2) as an impossibility which does not now follow from the defended sentence, even though it would follow by natural consequence. In a disputation where a conceptual impossibility must be defended, one need not admit all natural consequences, but only those which are “so manifest that their opposite cannot be an opinion”. The meaning of this principle is partly clarified by what Burley says about the acceptable impossibilities. They are such that do not imply contradictions by consequences belonging to a restricted class: According to Burley, one should not defend impossibilities that formally include opposites (*impossibile formaliter includens opposita*). ([16]) Burley’s use of the word *formaliter* here is noteworthy. It seems that the word is chosen to signify the strictest kind of impossibility, just as it begins to be used to signify the strictest validity of consequences. It seems that his choice of word is connected to the role this term gets in signifying the strictest validity of consequences: They begin to be called formal. Given that Burley gives the

¹⁷For a discussion about the author, see [33]. The texts by Master W and Walter Burley are edited in [16].

¹⁸“... ‘animal’ non sit in intellectu ‘hominis’.”([16])

principle in a context where “man is not an animal” is an acceptable *positio impossibilis* and thus does not include opposites “formally”, it seems that Burley does not here count a genus-species relation or perhaps any relation between mere terms as a formal relation in this sense. Formal consequences in the strictest sense are based on the structures of the sentences, not on the relations between the terms.

3 William Ockham

When we turn to the *Summa logicae* (from the 1320s) of Burley’s famous contemporary, William Ockham (1285–1347), we find some similar ideas further developed first in its sections about obligational disputations. Ockham formulates the characterization of the marked class of valid consequences slightly differently. According to him, in a *positio impossibilis* one is allowed to use only such consequences which are based on propositions and general rules that are known *per se*. The relevant text runs as follows:

not any impossible sentence whatever is to be admitted, because such an impossible sentence which manifestly implies contradictories in any intellect is not to be admitted (*non quaelibet propositio impossibilis est admittenda, quia illa propositio impossibilis quae manifeste apud omnem intellectum infert contradictoria non est admittenda*). Thus only such impossible propositions from which one cannot *infer contradictions by rules and propositions known per se*, which no intellect can doubt, are to be received as impossible positions.

([38], p. 739; *Summa logicae*, III-3, c. 42; *emphasis ours*)

Just after this characterization Ockham refers to “natural and simple” consequences in the manner of Burley’s treatise on consequences and almost in the manner of earlier obligation treatises. However, his description of the acceptable inferences is different from Burley’s natural consequences. In fact, it is much more reminiscent of Ockham’s own account of *formal* consequence in the section of *Summa logicae* devoted to various classifications of valid consequences. Let us now look at that text, where naturalness, in its turn, does not appear at all. ([38], pp. 587–590; III-3, c. 1)

Ockham, like Burley, begins his distinctions among valid consequences with the difference between those valid simply and those valid only *ut nunc*. Then he gives the distinction between validity by an intrinsic medium and validity by an extrinsic medium. (The words *medium* and *locus* are here used interchangeably, as the case often is, but the lines between intrinsic and extrinsic do not quite coincide in Ockham and in Burley.) Ockham first gives an example of the former type, a consequence valid by an intrinsic medium. Thus,

- (II) (1) Socrates does not run
 (2) A man does not run

According to Ockham, this inference is valid “in virtue of the medium ‘Socrates is a man’, and thus if this were not true: ‘Socrates is a man’, the consequence would not be valid.” Here “Socrates is a man” is the medium sentence guaranteeing the validity of the consequence. Ockham calls it “intrinsic” because it is formed of the *same terms* which occur in the consequence itself.

The example of validity by an extrinsic medium is equally interesting:

- (III) (1) Only a man is a donkey
 (2) Every donkey is a man

The validity of (III) does not rest on the truth of any propositions concerning men and donkeys, but it “is based on this general rule: ‘an exclusive proposition and a universal one with transposed terms signify the same and are mutually convertible’.” ([38], p. 588; III-3, c. 1) Furthermore, Ockham continues: “All syllogisms hold by such mediums.” In brief, some mediums are intrinsic to given terms, others “concern these terms no more than others” (ibid.); *intrinsic* mediums concern precisely the elements of the given particular inference, *extrinsic* mediums do not.

After introducing intrinsic and extrinsic mediums, Ockham continues to the distinction between *formal* and *material* validity, but he does not define those notions. He only states that some formal consequences are valid by an intrinsic medium, some by an extrinsic medium.¹⁹ (III) seems to be his prime example of a formal consequence with an extrinsic medium, and (II) holds formally by an intrinsic medium. Applying this to our basic examples, it seems that Ockham would classify our basic example (A) as formally valid by an extrinsic rule, while the analytic examples (B) and (C) would be formally valid by an intrinsic medium. Burley regards all these cases as natural, that is, valid by an intrinsic *locus*, but Ockham’s thought is very different. There is a major difference in their notions of intrinsic connection: Burley understands it as a matter of propositional contents, while Ockham uses a less liberal, “quotational” notion which demands the occurrence of the same terms in intrinsically connected propositions. This difference is characterized by Christopher Martin as a transition from “natural” to “formal” consequences as the logically privileged class. ([21])

What consequences are material, then? Unfortunately, there are textual prob-

¹⁹“Another distinction is that some consequences are material and some are formal. There are two kinds of formal consequences, since some are valid in virtue of an extrinsic medium which concerns the *form of the proposition*. ... Some are valid immediately in virtue of an intrinsic medium and mediately in virtue of an extrinsic medium ...” (*Summa logicae*, III-3, 1.) Note that some reference to the forms of the propositions is necessary in any case. As Ockham himself remarks, *mediately* the validity holds in virtue of extrinsic rules. Thus in (II), a general syllogistic rule justifies the consequence after the proper intrinsic medium has first been found out.

lems in exactly those sentences of *Summa logicae* which give the criteria for this.²⁰ Nevertheless, let us suggest the following interpretation: A consequence is materially valid if it holds in virtue of some general maxim concerning the truth, possibility or necessity of propositions, *and* because its terms make that maxim applicable. Thus (D) holds materially because of the maxim “*ex impossibile quodlibet*”, and because its terms make the premise impossible (men are necessarily not donkeys) and therefore the general maxim applicable. (E) is more problematic, because in traditional parlance it ought to be an extreme case of material consequence, but in fact it holds simply because “*ex impossibile quodlibet*”; conjunction of a proposition and its negation is impossible regardless of the particular terms. For this reason, we think Ockham could be forced to classify (E) as formally valid.

However, Ockham mentions one possible distinction which might still keep (E) material. Each of (A), (D) and (E) uses a medium that is in some sense extrinsic, not bound to the terms of the consequence. But the same passage makes clear that there are two kinds of such mediums, since Ockham especially mentions those which “concern the form of the proposition” (*respicit formam propositionis*) and contrasts them to mediums “concerning general conditions of propositions” (*respicens generales condiciones propositionum*). The medium that is followed in our basic example (A) is of the former kind, but the “*ex impossibile*” rule is not. Restriction to the first type of mediums would give a new sense of formality. Ockham, unfortunately, nowhere defines the *form* of proposition, and we cannot take for granted that he means the same thing with it as modern logicians do. Neither do we have any clear grasp of what he means with “general conditions of propositions”.

It seems that we can summarize the results for Ockham by considering the following question: How are we to show whether a sentence *q* follows from the sentence *p*? Ockham advises us to look first if there is a general rule that allows us to infer *q* from *p* directly (validity by extrinsic medium). Such rules may be based on either formal or material characteristics of *p* and *q*. If no rule of inference turns out to be applicable, we are to look for a suitable additional premise *r* so that the inference from *p* and *r* to *q* can be supported by some general rule of inference. The degree of validity depends on the logical or modal status of this additional premise (validity by intrinsic medium).

Both Ockham and Burley had a strictly systematic and calculative view of consequences. Though they belonged to opposing philosophical parties, Ockham a nominalist and Burley a strongly realist Scotist, they agreed in representing inferences by means of rules or mediums—intrinsic or extrinsic—the chain of which connects *p* and *q*. This approach is, in a way, completed in the detailed anonymous *Liber consequen-*

²⁰“Consequentia materialis est quando tenet praecise ratione terminorum et <cancel: non> ratione alicuius medii extrinseci respicientis praecise generales condiciones propositionum.” ([38], p. 589; *Summa logicae* III-3, 1) For the textual problems, see [30], p. 41, 76.

tiarum written soon afterwards, which brings together the results and classifications of both of them. It presents the whole theory by means of Ockhamist mediums but also lays down that they express inclusions of meaning.²¹

As regards formality, Ockham's novelty lies in his endeavour to connect his criteria to propositional structures, but his results are still far from clear. An Ockhamist *Tractatus logicae minor* gives a little more detailed account of formal validity than Ockham himself does in his great *Summa*.²² However, a satisfactory theory only becomes possible when it is made clear what the relevant notion of *form* actually is. This happened in the next stage, in two different directions. Then it also became customary to call the distinguished validity "formal", and the word "natural" was soon abandoned. ([21])

4 Jean Buridan

In order to find an explicit statement of a concept of formal validity based on *logical form* in the sense of the expression used nowadays we have to turn to the leading Parisian logician of the fourteenth century, Jean Buridan (c. 1300–1361?), and his *Tractatus de consequentiis* ([10], written after 1335). Buridan's full characterization of a valid consequence becomes elaborate, in order to block various abnormal counterexamples, but here it suffices to say that it is basically a refinement of the modal criterion that the consequence of q from p is valid if and only if $p \wedge \neg q$ cannot be the case. After discussing the notion of validity, Buridan examines the distinction between formal and material consequences. (The distinction between consequences valid simply and *ut nunc* comes only after that and is given much less weight.) According to Buridan, "a consequence is formal if every proposition with the same form would, if stated, be valid". Correspondingly, a valid consequence is material if, for some terms, a consequence in the same form would not be valid.²³

The idea is that all constituents of a proposition can be divided in two classes: Those belonging to its form and those belonging to matter. An inference is formally valid if all uniform substitutions for the material parts of the propositions still yield valid inferences. This is the same *substitutional* account of logical validity which is familiar to us also from modern logic. It was reinvented by Bolzano in his *Wis-*

²¹The treatise is edited in [30]. See especially chapter 1, and Schupp's comments.

²²See [12], especially pp. 78–80.

²³"A consequence is called *formal* if it is valid *in all terms*, keeping the same form. Or, speaking explicitly about the meaning of words, a consequence is formal when every proposition of the same form (*similis in forma*) which might be formed would be a valid consequence, like 'an A is B , thus a B is A '. A consequence is *material* if not every proposition of the same form would be a valid consequence, or, as is often said, if it does not hold in all terms, keeping the same form; for instance 'a man runs, thus an animal runs', because it does not hold in these terms: 'A horse walks, thus a tree walks'." ([10], pp. 22–23; *Tractatus de consequentiis* I, 4.)

senschaftslehre in 1837 and later developed by Russell and Tarski. ([9], §§ 147 and 148)²⁴

Buridan's first example of a formal consequence resembles our basic example (A). This is not surprising, but what is more remarkable is that his first example of a material consequence does not resemble the *ex impossibile* examples (D) or (E); instead, it is similar to the analytic example (B), which was until the 1320s traditionally classified into the logically privileged class as natural (sometimes also called "formal") because of the strong connection between the terms "man" and "animal". Thus his definition via logical form draws the line between formal and material inferences completely differently from Burley, and also Ockham. However, when Buridan explains the validity of consequences, he uses Ockham's method of additional premises, though he does not call them mediums. According to Buridan, the analytic example (B) can be *completed* to a formal consequence by including an additional premise, the necessary proposition "every man is an animal". Then the resulting inference will be syllogistic and clearly formal. In fact, he claims that material consequences are always "enthymematic": They rest upon implicit premises which can be spelled out if the inference must be shown valid. In this sense every consequence that is valid generally and not only *ut nunc* is reducible to the formal kind.²⁵

Ockham gave no account of the division between formal and material features of a proposition. He would already have had good conceptual means for that, since his semantic theory was based on a difference between significative categorematic terms, that is, the so-called absolute terms, and non-significative syncategorematic terms, which perform various functions on the categorematic terms of the sentence. Buridan, then, relies on this distinction in his semantics even more clearly than Ockham, and he also applies it in logic to determine the form and the matter of a proposition. Categorematic terms constitute the matter, the combination of syncategorematic terms constitutes the form. (See [10] I, 7.) Syncategorematic terms function to combine, to negate, and to yield various modes of reference or supposition for the categorematic terms in the proposition. In Buridan's listing of syncategorematic elements, we find propositional connectives, the copula of predication, negations, signs of quantification, and modal operators, among others. The number and order of terms and other

²⁴Exactly speaking, Bolzano defines the "universal validity" of a proposition by means of "arbitrary variations" of terms. This is the starting-point to Tarski's definition of logical truth, which is standard in present textbooks.

²⁵"And it seems to me that no material consequence is evident in inferring except through its reduction (*reductio*) to a formal one. Moreover, it is reducible to a formal one by adding some necessary proposition or propositions which, together with the assumed antecedent, render the consequence formal." ([10], p. 23; *Tractatus de consequentiis* I, 4) Or as Carnap states an analytic truth like "If Jack is a bachelor, then he is not married" is turned to a logical truth (or "L-truth", as he calls it) by introducing meaning postulates, like " $(x)(Bx \rightarrow \neg Mx)$ " for the predicate B. ([14], Supplement B, § 2) See also [22], pp. 75–77.

such things are relevant to the form of the proposition as well.²⁶ It is obvious that Buridan is working with what is nowadays commonly called the *logical form*.

Buridan studies also examples like (D) and (E) as interesting special cases. He first concludes that they are valid consequences: Anything follows from the impossible and the necessary follows from anything. A little later, he remarks that anything follows from the conjunction of two contradictories by a formal consequence. That is, (E) must be formally valid for Buridan. Furthermore, he notes that although (D) is only materially valid, it can be converted to a formal consequence by adding a necessary proposition as a second premise, and this added necessity should be simply the negation of the first premise, thus creating a complete formal contradiction as the premises. So the difference between (D) and (E) directly exemplifies the general classification of consequences into material and formal ones.

Shortly put, Buridan's definition of formal and material validity distinguishes the basic examples (A) and (E) as formal as they stand, while cases (B), (C) and (D) are material and need additional premises. Thus, his division is different from earlier definitions, including those by Burley and Ockham. Earlier authors saw (A), (B) and (C) as logically privileged (natural or formal), while (D) and (E) were thought to be accidental or material. Simplifying, we might say that Ockham established the terminology but only Buridan established the substance of the formal-material distinction.

The new definition of Buridan marks a genuine change. He saw formal consequences from a syntactic point of view, and other valid consequences appeared then as variants of formal ones, as variants with suppressed premises. This Buridanian model is also especially lucidly set out by Pseudo-Scotus in his remarkable commentary on *Prior Analytics* around 1350.²⁷ But apparently only rather few logicians, such as Buridan's pupil Albert of Saxony in his *Perutilis logica*, were willing to adopt Buridan's formulation of the substitutional account as the complete explanation of consequence.²⁸ Most authors wanted to pursue something less "schematic". According to their intuitions, a formal consequence must hold because of what the sentences

²⁶“Thus we hold that the copulas of categorical as well as of hypothetical [i.e. molecular] propositions pertain to the form, and similarly negations and signs, the number of propositions and of terms, and the mutual order of all these, and the relations of the relative terms, and the modes of signification regarding the quantity of propositions ...”. ([10], p. 30; *Tractatus de consequentiis* I, 7)

²⁷“A formal consequence is that which *holds for all terms*, when the disposition and form of the terms remain the same. And in the foregoing, we call ‘terms’ the subjects and the predicates of propositions, or parts of the subject or the predicate. Relevant to the form of the consequence are all the syncategorematic [elements] posited in the consequence, like conjunctions, signs of universality or particularity, negations and suchlike.” (Pseudo-Scotus, *Questions on Prior Analytics* I, q. 10; tr. Yrjönsuuri in [27], 229.) For some additional suggestions, see also [6] and [20], pp. 277–279.

²⁸As Albert says, “a consequence is called formal when every proposition of a similar form would be a valid consequence, if it were formed”. He even echoes Buridan's examples. ([3], p. 24r, *Perutilis logica* IV, 1.)

mean, and not just because of the structures of the sentences themselves. Indeed, it is remarkable that Buridan had achieved a new view, but it is also remarkable that this view was often consciously and deliberately rejected.²⁹

For instance, Ockham's close disciple Adam Wodeham says that there are two manners of defining formal consequences.³⁰ About the first fashion, he succinctly states that it accepts such modes of arguing which are "valid with all terms" (*teneat in omnibus terminis*). The second definition counts as formal even all the consequences which are valid "by an intrinsic medium which is unconditionally necessary" (*per medium intrinsecum simpliciter necessarium*). In fact, these two definitions are those by Buridan and Ockham, respectively. Wodeham wishes to claim the superiority of the Ockhamist definition over the substitutional one. He underscores such necessary truths that are, according to Aristotelian terminology, *per se primo modo*. They express the strongest metaphysical or conceptual necessities. As Wodeham notes, Buridan's definition does not respect such necessities. Because our examples (B) and (C) are not formally valid in Buridan's sense, correspondingly in this regard "the opposite of the consequent is consistent with the antecedent" (*oppositum consequentis stat cum antecedentis*).³¹ Wodeham disagrees. He thinks that renouncing an inference like (C) is always really as bad a contradiction as accepting propositions like "man is not man" or "God is not God". In another context, Wodeham explains his thought by referring to a terminological choice by Duns Scotus which seems to anticipate a lot of the later debates: Wodeham has his view because he looks at the conceptual relation that can be expressed with the phrase "to include formally" (*includere formaliter*). This relation must be understood with reference to the essences of the things at issue ([39], p. 298; *Lectura secunda*, dist. 6, q. 1, § 13.).³² Thus, (B) is valid because the

²⁹Our impression is thus different from that of, for example, [22], § 14. There it is said that "some of the English logicians" rejected the Buridanian view because of psychological considerations, whereas "most of the authors" did not think of the meanings of terms. (p.69, p.71) We are inclined to suppose that this was not the situation, but the issue calls for further study. In this respect, [29] opens very interesting new directions.

³⁰"I answer: It can be understood in two ways that a consequence is formal. *First*, that a syllogistic mode of arguing holds with all terms. ... *Second*, when in a special matter the consequence holds by an unconditionally necessary intrinsic medium." ([39], p. 83; *Lectura secunda*, dist. 2, q. 3, § 12.) See also p. 299, where Wodeham claims that in the sentence "infinite wisdom is formally infinite goodness" the word "formally" (*formaliter*) is to be understood to have the same force as the phrase "*per se primo modo*".

³¹"Further, I grant that as for the general form which can indifferently concern whichever terms, the opposite of the consequent stands with the antecedent." (ibid.)

³²Wodeham's quote from Scotus (*Ordinatio* I, dist. 8, pars 1, q. 4, n. 192–193) contains the phrase '*includere formaliter*', and Scotus's own discussion of formal relations has some connection to the formal validity of consequences, but it is even more implicit than in Wodeham. Wodeham's formal inclusion can be called analytic implication. As [15, 28, 29] explain, there was an opposition between two traditions: the English school, concentrating on analytic relations, and the Parisian school, concentrating on substitutivity.

concept “man” *formally includes* the concept “animal”, since “animal” is included in the real definition of “man”.

In the second half of the 14th century, a great number of authors base their examination of formal consequence on the requirement of such a formal dependence. In addition to Scotus and Wodeham, their language refers back to Walter Burley and beyond. Burley demanded of natural consequences that the antecedent must include the consequent. Such inclusion had often been presupposed already in the thirteenth century, but then it was thought to obtain between the understanding (*intellectus*) of the antecedent and of the consequent. In this form, the requirement can be traced back at least to Abelard, for whom it was even a criterion for the validity of a consequence, not only for its naturalness.

The crucial concept here is *intellectus*, which is usually translated as “understanding”, but can also mean almost the same as “concept”. The way it is here used is problematic. Some authors demand that the understanding of the consequent *q* is included in the understanding of the antecedent *p* (*est in intellectu antecedentis*), or with another translation, that *q* is conceptually included in *p*. Probably this has in fact the same sense as the alternative frequent phrase: That *q* is understood in *p*. Thus, Richard Billingham says simply that the consequent is understood in the antecedent, whereas most authors add another difficult term, asserting that the consequent is *formally* understood in the antecedent. Generally, this phrase “*formaliter intelligitur*” becomes a standard formula. Robert Fland says (*Consequentiae*, 1): “There are general rules for recognizing when a consequence is formal. The first one is this: When the consequence is *understood in* the antecedent formally.” ([32], p. 57) And we meet the same requirement in Ralph Strode’s famous logic, in Richard Lavenham, William of Osma, and others.³³

The condition of inclusion in the formal understanding of the antecedent is certainly open to interpretations. As we see the situation, its main import is that the connection between the two propositions must be because of their related *conceptual contents*. Nowadays, philosophers might say that the dependence must hold because of the meanings of expressions. Medieval logicians mainly used more essentialist language, but in any case, their “*intellectus*” comes close to conceptual content. Because of what belongs necessarily to what is spoken of in the antecedent *p* and in the consequent *q*, things cannot be as *p* signifies unless they also are as is signified by *q*.

This account obviously takes all *analytic* consequences, like our (B) and (C), as formal (neglecting some minor assembly work, such as that connected to the logical

³³For Lavenham, see [31]. For Osma, see [37]. For further discussion and more references, see [8], 155–156. Especially Strode’s logic was commonly used on the Continent. Strode writes according to [8]: “A consequence is sound by form (*de forma*) when, if the way in which facts are adequately signified by the antecedent is understood, the way in which they are adequately signified by the consequent is also understood; for instance, if anyone understands that you are a man, he will understand also that you are an animal.”

functions of negation). The resulting theory resembles that found in Burley's *De puritate artis logicae*. So, there is no extraordinary class of logical consequences among formally valid ones, and thus (A) is not of a different nature from (B) and (C). Moreover, the class of material consequences contains only the notorious cases of inferring from impossible or to necessity (both (D) and (E) are indistinguishably material). Logicians agreed that $p \wedge \neg q$ is impossible in a valid consequence from p to q , but the disagreement concerned the explanation of this. Buridan reduced it to his own notion of formality, defined in terms of the forms of propositions, whereas many of his contemporaries derived it from conceptual connections that hold because of the forms of things—as Scotus had already pointed out.

5 From the 1390s Onwards

The next step forward was taken when people saw that the two criteria of formal validity can perhaps be introduced to the same full account of consequences. Each criterion yields its own classification and thus for a complete picture a logician should know how to use both classifications. Even logicians who understood consequence as conceptual inclusion began to find the class defined with the structural criterion interesting. Thus, a member of the Oxford school, John Venator, quotes the substitutional definition of formal consequence. A consequence is called formal when it “holds in all terms”, that is, with all categorematic substitutions; it is material when it is grounded on relations of the categorematic terms in their significations. Thus, (B) is “good and material” (*bona et materialis*). Furthermore, Venator distinguishes between our cases (D) and (E), recognizing that a variant of (D), “this man is a donkey, therefore, this man is not a man” is valid but not formal. ([36], pp. 69–74)

Paul of Venice (1369–1429) introduces both definitions explicitly. He first defines a valid consequence with the usual requirement that the opposite of the consequent is repugnant to the antecedent. Many of the valid consequences are even *formal*. In Paul's *Logica parva*, the formal validity of a consequence from p to q means that the conjunction $p \wedge \neg q$ is not “imaginable to stand simultaneously without contradiction”. And the consequence is material if $p \wedge \neg q$ is impossible but still imaginable without any contradiction. ([24], p. 167 [65]; *Logica parva* III, 1) Here we can see that the development of *modal* theories has led to a distinction between different kinds, or perhaps grades, of necessity in consequences. Indeed, in the huge *Logica magna* of Paul of Venice, written just about 1400, we find even three levels of formality of consequences, based on (i) natural necessities, (ii) supernatural necessities, which cannot be broken even by God's power, and (iii) conceptual necessities, whose opposites are not even intelligible or imaginable.³⁴ As an example of the strongest

³⁴“A formally valid inference ... may be defined as one in which the contradictory of its conclusion, when it signifies primarily in the customary way, would be formally incompatible with the premiss

group, called *consequentia formalissima*, Paul gives “you are a man, therefore you are an animal”, which is like our (B).

For Paul of Venice, material consequences are only those with a non-conceptually impossible antecedent or a non-conceptually necessary consequent. According to him, examples like (D) are material, but (A), (B), (C), and (E) are all formal. Here the difference between (D) and (E) can look somewhat surprising, but Paul indeed provides a proof that “from an impossible proposition, which formally implies a contradiction, anything whatsoever [i.e. any proposition whatsoever] follows formally”. The example of such impossibility which implies formally everything is “You are other than yourself”. Paul seems to think here that a strictly formal consequence rests upon a special necessity which propositions like his standard example of necessity, “God exists”, do not possess. This proposition surely follows from any premise, but only materially; for it would not follow, “if it were contingent”, as Paul puts it. Apparently such a supposition is not feasible at all for conceptual necessities.³⁵ ([25], pp. 93–95; *Logica magna* II-4, ch. De rationali, ii)

Furthermore, Paul divides the formal consequences into those valid *de materia* and those valid *de forma*. This double distinction leads to such qualifications as “formally formal” or “formal *de forma*”. Such manner of speaking has been found curious or confused, but a comparison to Buridan can possibly shed light on it. Actually, the class of consequences valid *de forma* is defined purely substitutionally, via the *logical form* of the propositions. A consequence is valid *de forma*, if every consequence of the same form is valid.³⁶ The idea of substitutions in a constant logical form is the same as with Buridan. However, Paul’s notion of logical form is not equally lucid, since logical and semantic categories are not kept strictly separate: thus the inference from an inferior term to a superior one, from species to genus, comes to be taken as valid *de forma*, since it holds because of “the arrangement or relationship of the parts [of the proposition]”.³⁷ This means that though our (C) would obviously be *de materia*, (B) would still be *de forma*. A similar suggestion can be found from the

of that inference.” ([25], p. 89; *Logica magna* II-4, ch. De rationali, ii) “I call two things mutually incompatible in the highest degree when one of them cannot without contradiction be *understood or imagined* along with the other ...”. (p. 92)

³⁵It must be added that actually (D) is not a good example of Paul’s position. He would say that even it is formal within non-divine discourse, where man and donkey are incompatible; a completely material consequence needs a simply natural impossibility as its premise.

³⁶“An inference which is valid because of its form (*de forma*) may be defined as one where every inference of the same form is valid. ... An inference which is valid because of its matter (*de materia*) may be defined as one where not every inference of the same form is valid; for example, ‘only a father exists, therefore not only a father exists’. This is valid because of its matter, since not every inference of the same form is valid; for ‘only one thing exists, therefore not only one thing exists’ is not valid.” (p. 104; *De rationali*, iii.) See also *Logica parva* III, 1; pp. 168–169 [66–67].

³⁷In other words, the semantic structure of species and genera is allowed to contribute to the propositional form.

Ockhamist *Elementarium logicae*, but of course not in Buridan.³⁸

Among the later treatments of consequences, let us mention Paul of Pergula (d. 1451). Usually he follows Paul of Venice closely, but here he makes a little change. It seems as if he tried to replace the somewhat vague condition of “unimaginability” with something more definite and unambiguous. According to his explication, q follows formally from p , if p and $\neg q$ can “*nec realiter nec conceptibiliter*”, neither really nor *conceptually*, be together without manifest contradiction.³⁹ Stated in this way, the demand certainly seems less “psychological”. Then he continues by making a second distinction which demarcates the validity *de forma* with a highly restrictive condition of similarity of form.

A remarkable feature in the double distinction of the two Pauls is that it outlines the class of consequences which are formal but not *de forma*. This class had been vaguely anticipated in some earlier texts, such as the obligational treatises about defending impossibilities and the above-mentioned Wodeham text. Its precise composition was not quite clear even with the double distinction, because the notion of logical form contained obscurities separating (B) and (C). Anyway, it seems that during the fifteenth century it became customary to mention both criteria, but the criterion of validity *de forma* was often regarded as the most accurate logical characterization of formal validity. The same had been Buridan’s position, but he had no trouble with material conceptual truths since his systematic vision was that they could all be reduced to formal ones. Such an idea of a general elimination of non-logical analytic consequences by strict meaning postulates disappears in the fifteenth century.

The authors who used a double criterion of formality, like Paul of Venice and his followers, were in fact trying to define two different logical properties of inferences, and they seem to have been rather well aware of this situation. What most interested them, as the best subject of logical investigation, was the intersection of the classes defined by the two criteria, in other words, the *de forma* formal inferences. It is interesting that some authors really stated that only this intersection was the proper class of formal consequences, giving the definition that a consequence is formal just if the consequent is included in the antecedent *and* the consequence holds in all adequate substitutions. (Thus in the anonymous Spanish treatise edited in [34].⁴⁰) Taken by

³⁸See [13], p. 255. In this earlier Ockhamist text such consequences are still called formal and not *de forma*: “Thus, a consequence is called formal when it holds because of the form of the proposition, that is, if it holds whenever such a form of reasoning is used, like [for example] the consequence from inferior to superior without distribution is a good consequence.”

³⁹“Those are said to be formally repugnant which can neither really nor conceptually stand simultaneously without manifest contradiction, like: You run and do not move.” ([23], p. 87; *Logica* IV, 2)

⁴⁰“A formal consequence is such a valid consequence that any consequence of the similar form or [similar] arguing mode, when formed, is valid in whatever terms, and its consequence is from the understanding of its antecedent.” ([34], p. 206; *De consequenciis* II, 9)

itself, such a compound criterion sounds odd, but it fits well into the general picture of the two senses of formality.

6 Conclusion

For clarity, let us draw a chart of the alternative classifications for our five basic examples:

Burley	A, B, C natural A, B, E formal	D, E accidental C, D material
Ockham	A, B, C formal (A external;	D, E (?) material B, C internal)
Buridan	A, E formal	B, C, D material
Paul	A, B, C, E formal A, B, E <i>de forma</i>	D material C, D <i>de materia</i>

This list perhaps illustrates that the authors were attempting to capture several different intuitive ideas. Therefore their classifications became so various. Two main problems seem to pertain to the characterization of valid consequences by the authors we have discussed. *First*, there is the treatment of the so-called paradoxes of implication. The standard definition of consequence obviously gives the result that every argument with an impossible premise or a necessary conclusion must be valid—which is perhaps not “paradoxical” but certainly not intuitive, either. It seems rather clear to us that all the logicians discussed did accept consequences like our examples (D) and (E) as valid. That is, the rules “anything follows from the impossible” and “the necessary follows from anything” were approved as correct. However, most authors wanted to distinguish such consequences from those where the inference was based on some kind of connection between the premises and the conclusion. Furthermore, it can well be of some importance that logicians often saw a difference between (D) and (E): Formal contradiction was frequently taken to be conceptually separate from impossibility, even though both were sufficient to warrant inferences.

The problem of conceptual content becomes still more clearly visible in the *second* main issue in the distinctions of consequences. It derives from the notion of “*intellectus*”. Criteria of formal inference could be based on it, and such criteria became comparatively popular and explicit in England in the latter half of the fourteenth century. Many modern scholars have labelled these criteria as “psychological” or “epistemic”. As we have seen, this use of the concept was not a fourteenth-century innovation, and it seems that it was reintroduced in order to provide a viable alter-

native to Buridan's definition of formal validity based solely on propositional logical forms. Many logicians were not satisfied with so strict a criterion of formal validity, because they felt that it unjustly excluded important valid inferences from the class of formally valid inferences. The necessity on which inferences like (B) and (C) are grounded looks rather strong—although it is of different kind from that found in (A).

It is no surprise if some authors wish to employ psychological terminology and describe conceptual necessities in terms of what it is possible to think. However, it is also true and natural that many authors use terms like "*intellectus*" without any psychological overtones. Conceptual conceivability or imaginability need not be dependent on psychological capacities, and in fact, for fourteenth-century authors "imaginable" usually did not mean something which is manageable for human or psychological faculty of imagination in practice. Rather, imaginability was often discussed as something that restricted already God himself in his creation. Such considerations can hardly be called psychological.

It seems to us that the best way of describing what we are dealing with in these doctrines of conceptual conceivability and conceptual content is to connect them to *analyticity*: analytic truth and analytically valid inferences. Such subjects had their essential influence on consequence theory, although medieval—quite like modern logicians—could not reach a fully clear and satisfactory definition of analytic truth. Then, as we see it, the fourteenth century saw an innovation when an explanation of formal validity was sought from the concept of logical form. This, in its turn, corresponds to our present concept of *logical* validity. The innovation was first rejected by many authors, and it would be overly simple to connect this reaction straightforwardly to views about human psychology or naturalist epistemology. Rather, the issue that caused worry was conceptual truth and validity based on conceptual connections. And an interesting feature in the late medieval consequence theory is precisely the fashion in which the idea of logical form and the idea of analyticity develop in interaction. The latest period, from Paul of Venice onwards, seems to be the time when the difference and relation of analytic validity and logical validity was consciously explicated and analysed for the first time.

References

- [1] Abelard, Peter, 1956, *Dialectica*, Edited by L.M. De Rijk, Wijsgerige teksten en studies, Assen: Van Gorcum.
- [2] T. Aho, 2011, "Consequences, theory of", in H. Lagerlund (ed.), *Encyclopedia of Medieval Philosophy*, Dordrecht: Springer.
- [3] Albert of Saxony, 1974, *Perutilis logica*, Venetiis 1522, repr. Hildesheim: Georg Olms Verlag.

- [4] J. Archambault, 2018, “Consequence and formality in the logic of Walter Burley”, *Vivarium*, **56(4)**: 292–319.
- [5] E. J. Ashworth, 1974, *Language and Logic in the Post-Medieval Period*, Dordrecht: D. Reidel.
- [6] J. Bendiek, 1952, “Die Lehre von den Konsequenzen bei Pseudo-Scotus”, *Franziskanische Studien*, **34**: 205–234.
- [7] I. Boh, 1982, “Consequences”, in N. Kretzmann et al. (eds.), *The Cambridge History of Later Medieval Philosophy*, pp. 300–314, Cambridge: Cambridge University Press.
- [8] I. Boh, 2001, “Consequence and rules of consequence in the Post-Ockham Period”, in M. Yrjönsuuri (ed.), *Medieval Formal Logic. Obligations, Insolubles and Consequences*, pp. 147–181, Dordrecht: Kluwer.
- [9] Bolzano, Bernard, 1929–1931, *Wissenschaftslehre*, Leipzig: Felix Meiner.
- [10] Buridan, Johannes, 1976, *Tractatus de consequentiis*, Edited by H. Hubien, *Philosophes médiévaux* 16, Louvain: Publications universitaires.
- [11] Burley, Walter, 1955, *De puritate artis logicae tractatus longior; with a revised edition of the Tractatus brevior*, Edited by Ph. Boehner, Franciscan Institute Publications, Text Series 9, St. Bonaventure: Franciscan Institute.
- [12] E. M. Buytaert, 1964, “The tractatus logicae minor of Ockham”, *Franciscan Studies*, **24**: 34–100.
- [13] E. M. Buytaert, 1965, “The elementarium logicae of Ockham”, *Franciscan Studies*, **25**: 151–276.
- [14] R. Carnap, 1947, *Meaning and Necessity*, Chicago: University of Chicago Press, Enlarged edition 1956.
- [15] C. Dutilh Novaes and M. Crimi, 2024, “Medieval theories of consequence”, *Stanford Encyclopedia of Philosophy*.
- [16] R. Green, 1963, *The Logical Treatise ‘De obligationibus’: An Introduction with Critical Texts of William of Sherwood (?) and Walter Burley*, PhD thesis, Louvain.
- [17] N. J. Green-Pedersen, 1984, *The Tradition of the Topics in the Middle Ages: The Commentaries on Aristotle’s and Boethius’ ‘Topics’*, München: Philosophia Verlag.
- [18] R. Kilwardby, 2015, *Notulae libri Priorum*, Oxford: Oxford University Press.
- [19] P. King, 2001, “Consequence as inference: mediaeval proof theory 1300–1350”, in M. Yrjönsuuri (ed.), *Medieval Formal Logic. Obligations, Insolubles and Consequences*, pp. 117–145, Dordrecht: Kluwer.
- [20] W. Kneale and M. Kneale, 1962, *The Development of Logic*, Oxford: Oxford University Press.
- [21] C. J. Martin, 2018, “The theory of natural consequence”, *Vivarium*, **56(4)**: 340–366.
- [22] E. A. Moody, 1953, *Truth and Consequence in Mediaeval Logic*, Amsterdam: North-Holland Publishing Company.
- [23] Paul of Pergula, 1961, *Logica and Tractatus de sensu composito et diviso*, Edited by M. A. Brown, Franciscan Institute Publications, Text Series 13, St. Bonaventure: Franciscan Institute.
- [24] Paul of Venice, 1984, *Logica parva*, München: Philosophia Verlag.

- [25] Paul of Venice, 1990, *Logica magna, Part II, Fascicule 4*, Oxford: Oxford University Press.
- [26] Peter of Spain, 1992, *Synkategoreumata*, Edited by L. M. de Rijk, Studien und Texte zur Geistesgeschichte des Mittelalters 30, Leiden: E. J. Brill.
- [27] Pseudo-Scotus, 2001, “Questions on Aristotle’s *Prior Analytics* I, Question X”, in M. Yrjönsuuri (ed.), *Medieval Formal Logic. Obligations, Insolubles and Consequences*, pp. 225–234, Dordrecht: Kluwer.
- [28] S. Read, 2012, “The medieval theory of consequence”, *Synthese*, **187(3)**: 899–912.
- [29] L. Saario-Ramsay, 2026, “The two schools of formal consequence in the fourteenth century: redrawing the distinction”, *British Journal for the History of Philosophy*, **34**: 216–239.
- [30] F. Schupp, 1988, *Logical Problems of the Medieval Theory of Consequences*, Napoli: Bibliopolis.
- [31] P. Spade, 1974, “Five logical tracts by Richard Lavenham”, in J. R. O’Donnell (ed.), *Essays in Honour of Anton Charles Pegis*, pp. 70–124, Toronto: Pontifical Institute of Mediaeval Studies.
- [32] P. Spade, 1976, “Robert Fland’s consequentiae: An edition”, *Mediaeval Studies*, **38**: 54–84.
- [33] P. Spade and E. Stump, 1983, “Walter Burley and the *Obligationes* attributed to William of Sherwood”, *History and Philosophy of Logic*, **4(1)**: 9–26.
- [34] J. Spruyt, 1999, “A fifteenth-century Spanish treatise on consequences”, *Vivarium*, **37(2)**: 178–257.
- [35] E. Stump, 1989, *Dialectic and Its Place in the Development of Medieval Logic*, Ithaca: Cornell University Press.
- [36] Venator, Johannes, 1999, *Logica*, Edited by L.M. de Rijk, Grammatica speculativa 6, Stuttgart: Frommann-Holzboog.
- [37] Wilhelm von Osmā, 1991, *De Consequentis — Über die Folgerungen*, Hamburg: Meiner.
- [38] William of Ockham, 1974, *Summa logicae*, Opera Philosophica I, St. Bonaventure: Franciscan Institute.
- [39] Wodeham, Adam, 1990, *Lectura secunda in librum primum sententiarum, Volume II*, St. Bonaventure: St. Bonaventure University.
- [40] M. Yrjönsuuri, 2024, “Valid on formal grounds”, in C. G. Normore and S. Schmid (eds.), *Grounding in Medieval Philosophy*, pp. 251–280, Historical-Analytical Studies on Nature, Mind and Action, Cham: Springer, 10.1007/978-3-031-53666-3_12.

中世纪后期关于形式后承的观点

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摘 要

本文通过例子呈现出中世纪不同的对后承（*consequentiae*）分类为形式与实质后承，或自然与偶性后承的做法。中世纪是通过以下的一个模态标准来理解后承的有效性的，即在其后件不为真的情况下，其前件不可能为真。中世纪的逻辑学家们使用了两个主要的直觉的思想来提炼出一类具有优先地位的有效后承：（1）概念包含（后件的思想包含在了前件的思想中）；（2）通过替换原则来定义逻辑形式（即此后承针对任何词项都是有效的）。“形式的”（*formalis* 或 *de forma*）这个概念成为了一个针对此具有优先地位的有效后承类别的标准术语。类似地，“实质的”（*materialis* 或 *de materia*）被采纳用来指较低等类别的非形式的推理。大概是约翰·布里丹（John Buridan）发明的且由他所谨慎表达出的替换原则或许在许多现代逻辑学眼里是特别有吸引力的。但十四世纪的时候，它并未获得广泛的承认。它如被承认，也被认为是概念包含原则或现在我们说的分析有效性的另一个不同表达。

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