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Paradox of **Nothing**: A Paracomplete Solution*

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Abstract. The concept of nothing(ness) is a profound philosophical enigma, it is simultaneously a thing but no thing, and hence is paradoxical. To solve this paradox, Priest proposes a systematic formalization for theory of nothing based on paraconsistent logic and mereology. In this paper, we propose an alternative approach to resolve the paradox using a paracomplete logic, Łucasiewich 3-valued logic. We argue that, even though we accept Priest's characterizations of nothing, accepting contradictions about nothing is still not necessary.

1 Introduction

The concept of nothing(ness) is inherently enigmatic and embodies a paradox. It represents the ultimate absence, the point at which all things have been eliminated. However, in a peculiar twist, nothing paradoxically exists as a subject of discourse. We can talk about it, make statements about it, and even consider it in a philosophical context. This leads to the conundrum: nothing is simultaneously a thing but no thing. This is the paradox of nothing.

The debate surrounding the philosophical concept of nothingness is frequently contentious. The crux of this debate hinges on whether "nothing" or "nothingness" is employed as a noun, as opposed to being used as a phrase that quantifies. If it could be employed as a noun, what kind of object does it refer to? To answer this question, there are different ways to account for "nothing".

Voltolini ([8]) argues that "nothing" is not a genuine name, he consider it as a definite description, that is the thing satisfying the property of being a thing such that there is no thing that is identical to it. Using Russellian way to deal with definite descriptions, he concludes that nothing doesn't denote anything. Oliver and Smiley ([1]) makes a distinction between "nothing" as a quantifier and as an empty term. They use "zilch" as the empty term and define it as $\iota x x \neq x$. Since all things are self-identical, "zilch" denote no things. Simionato ([7]) considers "nothing" as a consistent object. He defines it in terms of empty possible worlds, and thinks that "nothing" is a world

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in which no thing exists. In [2, 3, 4, 5], Priest argues that, unlike the definition in [7], "nothing" is an inconsistent object, and it is the absence of everything but not the empty worlds. Unlike the definitions in [1] and [8], "nothing" denotes the object which is the mereological fusion of no things. It is not a empty denotation word. He constructs a pareconsistent based mereology to account for the theory of nothing.

In this paper, we will propose another approach, which is based on Łucasiewich 3-valued logic, a paracomplete logic, to solve the paradox of nothing. It shows that the paraconsistent logic is not a unique way to solve the problem even we accept the Priest's characterizations of nothing. Hence, accepting the contradiction is not necessary.

The paper is structured as follows. In Section 2, we will recall some preliminaries which will be used in the susequent sections. In Section 3, we present the Priest's approach, which is based on paraconsistent logic, for the theory of nothing. In Section 4, we propose our approach, which is based on a paracomplete logic, to solve the paradox.

2 Preliminary

In [5], Priest argues that nothing could be used as a noun, for example, "Hegel and Heidegger wrote about nothing, but they said different things about it". ([5, p.19]) But does nothing, as a noun, refer to an object?

From the perspective of Meinonism, Priest thinks that "an object is the kind of thing that can be referred to by a name, be subject to predication, be quantified over, be the target of an intentional state". In this sense, nothing is also an object. Since some objects exist and some do not from this point of view, "Contrary to the way that Kant is often—and erroneously—interpreted, exists is a perfectly ordinary monadic predicate."([5, p.27]) Not like the standard quantifiers (e.g. $\exists$) used in first-order logic, Priest introduces the general quantifiers (e.g. $\mathfrak{S}$) to the language to quantify over both existent and non-existent objects, and then the standard existential quantifier becomes a monadic predicate in the language. Hence "nothing is something" is formalized as $\mathfrak{S}x(x = \mathbf{n})$.[1]

Furthermore, nothing is not only a object but also a specific object. We could use some properties to characterize it and pick it out, that is, use some definite descriptions or indefinite descriptions to specify it.[2] According to Priest's definition, nothing is an inconsistent object, and it is the absence of everything intuitively. In order to make it more precise, he introduces the mereological concepts to define nothing as the fusion of no things. Mereology is the theory of parts and wholes. One of the central concepts

[1] In the following paper, we use boldface letters **nothing**, **everthing** to mean these words as nouns.

[2] The definite descriptions can be considered as a special indefinite descriptions, that is, the unique one.

of mereology is that of a mereological fusion, that is, the result of putting together a bunch of things to make a bigger thing. The converse of putting things together is taking them away. If you take something away, you have an absence. Therefore, in the formal language, he introduces the indefinite description and mereological operators to the language.

The language is a first-order language with identity, a binary predicate, and a general quantifier (instead of existential quantifier), which is defined inductively as below:

$$A ::= t < t \mid (t = t) \mid \neg A \mid (A \to A) \mid (A \wedge A) \mid (A \vee A) \mid \mathfrak{S}xA(x)$$

where t is a constant, or a variable, or a indefinite description with the form of $\varepsilon xA(x)$, $\mathfrak{A}xA(x) := \neg\mathfrak{S}x\neg A(x)$, and $A \leftrightarrow B := (A \to B \wedge B \to A)$. The formula $x < y$ means x is a proper part of y intuitively. In what follows, we will use $x, y, z \ldots$ (with or without subscripts) to denote arbitary variables, $a, b, c \ldots$ (with or without subscripts) and $A, B, C \ldots$ (with or without subscripts) to denote arbitary formulas, and use $t_1 \neq t_2$ as the abrreviation for $\neg(t_1 = t_2)$ in the language. Without bringing confusions, we will omit the parenthesis as many as we can. In order to express more mereological operations, we introduce the following abbreviations:

1. $x \leq y := x < y \vee x = y$
2. $x \circ y := \mathfrak{S}z(z \leq x \wedge z \leq y)$
3. $x \bullet y := \neg(x \circ y)$

$\circ$ is a overlap operation, the formula $x \circ y$ means there are some common parts between x and y, conversely, $\bullet$ means two objects haven't any overlap intuitively. In what follows, we will introduce the axiomatic system of the standard mereology, which is in fact mereological axioms plus the first-order logic, and the paraconsistent first-order logic first. Priest combines them together to formalize the theory of nothing.

The axiomatic system for mereology is the first-order logic (replacing "$\exists$" with "$\mathfrak{S}$") with identity, a binary predicate $<$, and the following characteristic axioms for mereology:

M1 $x < y \wedge y < z \to x < z$
M2 $x < y \to \neg y < x$
M3 $\neg y \leq x \to \mathfrak{S}z(z \leq y \wedge z \bullet x)$
M4 $\mathfrak{S}xA(x) \to \mathfrak{S}z\mathfrak{A}y(y \circ z \leftrightarrow \mathfrak{S}x(y \circ x \wedge A(x)))$

where M1-M3 are for characterizing the properties of "part-of" relation. But M4 is a principle which Priest assumes for general composition. It means that if there is some object (both existent or non-existent) satisfying the condition $A(x)$ charecterizes, then the merelogical fusion of them still constitutes an object. In the next section, we will see how he does use this principle to define nothing. To express the indefinite descriptions in this system, we also need to add the following axiom:

$$\textbf{M5}\ \mathfrak{S}xA \to A_x(\varepsilon xA)$$

in which $A_x(t)$ means replacing the occurrence x in A with t.

Let $\sigma xA(x) := \varepsilon z\mathfrak{A}y(y \circ z \leftrightarrow \mathfrak{S}x(y \circ x \wedge A(x)))$, that is, the mereological fusion of all objects satisfying the property A. Using M1-M5 (plus the first-order logic), we could prove the following corollaries:

Corollary 1. (1) $\mathfrak{S}xA(x) \to \mathfrak{A}y(y \circ \sigma xA(x) \leftrightarrow \mathfrak{S}x(y \circ x \wedge A(x)))$
(2) $\mathfrak{A}z(z \circ x \leftrightarrow z \circ y) \to x = y$
(3) $A(x) \to x \leq \sigma xA(x)$

Moreover, let $u - v := \sigma x(x \leq u \wedge x \bullet v)$ given that $\neg u \leq v$, we have:

(4) $(u - v) \bullet v$
(5) $y \circ u \to ((y \circ u - v) \vee y \circ v)$

we could consider $u - v$ as the relative complement of v with respect to u. Correspondingly, we can also define the absolute complement. Before doing this, Priest uses M4 to define the object **e** (means "everything") as $\sigma xx = x$. Intuitively, since all things are identical to themselves, the object "everything" is the mereological fusion of every thing (both existent and non-existent) indeed. [3] Then it follows that

(6) $\mathfrak{A}y\ y \leq \mathbf{e}$

The absolute complement $\bar{v}$ is the complement of v relative to **e**, which is defined as:

$$(7)\ \mathbf{e} - v := \mathfrak{A}v(\mathbf{e} \neq v \to \mathfrak{S}x(x \bullet v \wedge \mathfrak{A}y(y \circ x \vee y \circ v)))$$

By (2), the uniqueness of $\bar{v}$ follows. But notice that $\bar{\mathbf{e}}$ doesn't exist since there is no empty fusion.[4]

A logic is paraconsistent if the explosion law doesn't hold in the logic. The first-order paraconsistent logic **RM3** consists of the following axioms and rules[5]:

1. Axioms :

   A1 $A \to A$
   A2 $(A \wedge (A \to B)) \to B$
   A3 $(A \wedge B) \to A$
   A4 $(A \wedge B) \to B$
   A5 $((A \to B) \wedge (A \to C)) \to (A \to (B \wedge C))$

[3] In section 3, we will argue that only existent things can be claimed to be identical to themselves.

[4] In the next section, we will see how Priest modifies the standard mereology by admitting the empty fusion, and hence $\bar{\mathbf{e}}$ exists in his theory.

[5] It is a bit different from what Priest uses in [5, pp. 53–54]

A6 $A \to A \vee B$
A7 $B \to A \vee B$
A8 $(A \to C) \to ((B \to C) \to (A \vee B \to C))$
A9 $(A \wedge (B \vee C)) \to ((A \wedge B) \vee (A \wedge C))$
A10 $(A \to \neg A) \to \neg A$
A11 $(A \to \neg B) \to (B \to \neg A)$
A12 $\neg\neg A \to A$
A13 $(\neg A \wedge B) \to (A \to B)$
A14 $\neg A \to (A \vee (A \to B))$
A15 $\mathfrak{A}xA \to A_x(c)$
A16 $\mathfrak{A}xx = x$
A17 $A \vee \neg A$

2. Rules:

MP $A, A \to B \vdash B$
Adj $A, B \vdash A \wedge B$
Aff $A \to B, C \to D \vdash (B \to C) \to (A \to D)$
UG $A \vdash \mathfrak{A}xA$
Subst $a = b \vdash A_x(a) \leftrightarrow A_x(b)$

3 Theory of Nothing: a Paraconsistent Approach

In this section, we will introduce a theory of nothing which Priest proposes in [5, Part II]. He changes the base of standard mereology (classical first-order logic) to the paraconsistent first-order logic **RM3** above. Therefore, he could use an contradictory sentences to characterize the inconsistent property of **nothing**.

3.1 Axiomatic system PMn

As we see in Section 2, on the one hand, **nothing** can be used as a noun, we could introduce a constant **n** to denote it. On the other hand, **nothing** is a particular object, we could use some properties to pick it out. It can be characterized as the mereological fusion of no things. In order to formalize this definition, we need to introduce an unary property "no thing" and then the characterization of **nothing** follows by the principle of general composition. We have known that the property that "x is a thing (or object)" is symbolized as "$\mathfrak{S}yx = y$", and then "x is no thing" can be symbolized as "$\mathfrak{A}yx \neq y$" dually, which is equivalent to $x \neq x$. Therefore, by M4 and $\mathfrak{S}xx \neq x$,

$$\textbf{M4n} \ \mathfrak{S}z\mathfrak{A}y(y \circ z \leftrightarrow \mathfrak{S}x(y \circ x \wedge x \neq x))$$

Let **n** $:= \sigma x(x \neq x) = \varepsilon z \mathfrak{A} y(y \circ z \leftrightarrow \mathfrak{S} x(y \circ x \wedge x \neq x))$, then by M5 and M4n, it follows that

$$(8)\ \mathfrak{A} y(y \circ \mathbf{n} \leftrightarrow \mathfrak{S} x(y \circ x \wedge x \neq x))$$

Since (3) is not provable in RM3 based mereology, in order to say any object, which is no thing, is part of **nothing**, we also need to add the following axiom:

$$\textbf{M4n+}\ \mathfrak{A} x(x \neq x \rightarrow x \leq \mathbf{n})$$

Moreover, since (2) cannot be derived in RM3 based mereology, instead we add it as an axiom in the theory of **nothing**:

$$\textbf{M2+}\ \mathfrak{A} z(z \circ x \leftrightarrow z \circ y) \rightarrow x = y$$

Analogously, for characterizing **everything**, it is necessary to add two new axioms:

$$\textbf{M4e}\ \mathfrak{S} z \mathfrak{A} y(y \circ z \leftrightarrow \mathfrak{S} x(y \circ x \wedge x = x))$$

and let **e** $:= \sigma x(x = x) = \varepsilon z \mathfrak{A} y(y \circ z \leftrightarrow \mathfrak{S} x(y \circ x \wedge x = x))$,

$$\textbf{M4e+}\ \mathfrak{A} x(x = x \rightarrow x \leq \mathbf{e})$$

In the characteristic axiom of **n**, Priest assumes that the mereological fusion of empty exists, and hence, not like the complement in Section 2, we do not need to make distinction between relative complement and absolute complement since there is also the complement of **e**. Therefore the antecedent of (7) can be dropped, it is modified by the following axiom:

$$\textbf{Mc3}\ \mathfrak{A} x \mathfrak{S} y(x \bullet y \wedge \mathfrak{A} z((z \neq z) \vee (z \circ x) \vee (z \circ y)))$$

Let $Comp(x, y) := x \bullet y \wedge \mathfrak{A} z((z \neq z) \vee (z \circ x) \vee (z \circ y))$, and $\bar{x} := \varepsilon y Comp(x, y)$, in order to make all complements (both relative and absolute) unique, the following axiom is also necessary:

$$\textbf{Mc3+}\ Comp(x, y_1) \wedge Comp(x, y_2) \rightarrow y_1 = y_2$$

Definition 1 (PMn). The axiomatic system **PMn** consists of **RM3** and the following axioms:

$$\textbf{M1}, \textbf{M2}, \textbf{M2+}, \textbf{Mc3}, \textbf{Mc3+}, \textbf{M4n}, \textbf{M4n+}, \textbf{M4e}, \textbf{M4e+}, \textbf{M5}$$

The object **n** is inconsistent in the sense that it is both an object and not an object. It is also called the *paradox of nothing*, which can be formalized as $\mathfrak{S} xx = \mathbf{n} \wedge \neg \mathfrak{S} xx = \mathbf{n}$. In what follows, we can show that it is derivable in **PMn**.[6]

[6] For saving space, we omit some steps in the proof.

Proof.

1.	$\mathfrak{A}xx = x$	A16
2.	$\neg\mathfrak{S}xx \neq x$	1, Definition of $\mathfrak{A}$
3.	$\mathfrak{A}y(y \circ \mathbf{n} \leftrightarrow \mathfrak{S}x(y \circ x \wedge x \neq x))$	M4n, M5, MP
4.	$\mathfrak{A}yy \bullet \mathbf{n}$	2,3,UG
5.	$\mathbf{n} \bullet \mathbf{n}$	4, A15, MP
6.	$\mathbf{n} \circ \mathbf{n} \leftrightarrow \mathfrak{S}z(z \leq \mathbf{n} \wedge z \leq \mathbf{n})$	Definition of $\circ$
7.	$\neg\mathfrak{S}zz \leq \mathbf{n}$	5, 6, definition of $\leftrightarrow$
8.	$\neg\mathbf{n} \leq \mathbf{n}$	7, A15, MP
9.	$\mathbf{n} \neq \mathbf{n}$	8, De Morgan, A3, MP
10.	$x = \mathbf{n} \vee x \neq \mathbf{n}$	A17
11.	$\mathbf{n} \neq \mathbf{n} \to (x = \mathbf{n} \to x \neq \mathbf{n})$	HS,A11
12.	$x = \mathbf{n} \to x \neq \mathbf{n}$	9,11, MP
13.	$x \neq \mathbf{n} \to x \neq \mathbf{n}$	A1
14.	$(x = \mathbf{n} \vee x \neq \mathbf{n}) \to x \neq \mathbf{n}$	A8, 12,13, MP
15.	$x \neq \mathbf{n}$	10,14, MP
16.	$\mathfrak{A}xx \neq \mathbf{n}$	15, UG
17.	$\neg\mathfrak{S}xx = \mathbf{n}$	16, definition of $\mathfrak{A}$
18.	$\mathbf{n} = \mathbf{n}$	1, A15, MP
19.	$\mathfrak{S}xx = n$	18, A15, A11, MP
20.	$\mathfrak{S}xx = n \ \wedge \ \neg\mathfrak{S}xx = \mathbf{n}$	14, 16, Adj

□

Hence, from the perspective of proof theory, the paradox of nothing is admitted in **PMn**. In the subsequent subsections, it is also shown that there is a model which satisfies all axioms and rules in **PMn**.

It is also very interesting to prove that the complement of **e** is **n** in **PMn**. In fact, it gives another way to characterize **nothing**, that is, the absence of every thing. By definition, $Comp(\mathbf{e}, \mathbf{n}) := \mathbf{e} \bullet \mathbf{n} \wedge \mathfrak{A}z((z \neq z) \vee (z \circ \mathbf{e}) \vee (z \circ \mathbf{n}))$, which can be derived as follows:

Proof.

1.	$\neg\mathfrak{S}zz \leq \mathbf{n}$	7 in the previous proof
2.	$\mathfrak{A}z\neg z \leq \mathbf{n}$	1, Definition of $\mathfrak{A}$
3.	$\mathfrak{A}z\neg(z \leq \mathbf{n} \wedge z \leq \mathbf{e})$	2, A6, De Morgan
4.	$\mathbf{e} \bullet \mathbf{n}$	3, Definition of $\bullet$

5.	$\mathfrak{A}xx \leq \mathbf{e}$	M4e+, A16, MP
6.	$x \circ \mathbf{e}$	definition of $\circ$
7.	$(x \neq x) \vee (x \circ \mathbf{e}) \vee (x \circ \mathbf{n})$	6, A6, A7
8.	$\mathfrak{A}x((x \neq x) \vee (x \circ \mathbf{e}) \vee (x \circ \mathbf{n}))$	7, UG
9.	$\mathbf{e} \bullet \mathbf{n} \wedge \mathfrak{A}x((x \neq x) \vee (x \circ \mathbf{e}) \vee (x \circ \mathbf{n}))$	4, 8, Adj

□

3.2 Semantics for PMn

In this subsection, we will introduce the semantics for **PMn**, which can be found in [5, Chapter 4]. In order to solve the paradox of **nothing**, it is not enough to give a proof system only, but also to give a concrete model to make the system work.

Definition 2 (Structure). An interpretation for **PMn** without ε-terms is a pair $\mathcal{S} = \langle D, \delta \rangle$ which satisfies:

1. $\mathbf{n}, \mathbf{e} \in D$;
2. $\delta(c) \in D$ for any constant c;
3. $\delta(P) = (\delta^+(P), \delta^-(P)) \subseteq D^n \times D^n$, s.t. $\delta^+(P) \cup \delta^-(P) = D^n$ for any n-ary predicate P;
4. $\delta(=) = (\delta^+(=), \delta^-(=)) = (\{(d, d) \mid d \in D\}, \{(\mathbf{n}, \mathbf{n})\})$.

Intuitively, $\delta^+(P)$ is a set of n-tuples of D which is the extension of the predicate P, and $\delta^-(P)$ is a set of n-tuples of D which is the anti-extension of the predicate P. Although the union of $\delta^+(P), \delta^-(P)$ exhausts D^n, not like the classical first-order logic, the intersection of them is not necessary empty. Therefore, it is possible that some contradictions are admissible.

Let $\models^+$ and $\models^-$ stand for truth and falsity with respect to a structure $\mathcal{S}$ respectively, then the truth conditions for the language is defined inductively as follows: let P be an n-place predicate:

- $\models^+ P(c_1, \ldots, c_n)$ iff $\langle \delta(c_1), \ldots, \delta(c_n) \rangle \in \delta^+(P)$.
- $\models^- P(c_1, \ldots, c_n)$ iff $\langle \delta(c_1), \ldots, \delta(c_n) \rangle \in \delta^-(P)$.

- $\models^+ \neg A$ iff $\models^- A$.
- $\models^- \neg A$ iff $\models^+ A$.

- $\models^+ A \wedge B$ iff $\models^+ A$ and $\models^+ B$.
- $\models^- A \wedge B$ iff $\models^- A$ or $\models^- B$.

- $\models^+ A \vee B$ iff $\models^+ A$ or $\models^+ B$.

- $\models^- A \vee B$ iff $\models^- A$ and $\models^- B$.

- $\models^+ A \to B$ iff (if, materially, $\models^+ A$ then $\models^+ B$) and (if, materially, $\models^- B$ then $\models^- A$).
- $\models^- A \to B$ iff $\models^+ A$ and $\models^- B$.

- $\models^+ \mathfrak{A}xA$ iff for all $d \in D$, $\models^+ A_x(c_d)$.
- $\models^- \mathfrak{A}xA$ iff for some $d \in D$, $\models^- A_x(c_d)$.

- $\models^+ \mathfrak{S}xA$ iff for some $d \in D$, $\models^+ A_x(c_d)$.
- $\models^- \mathfrak{S}xA$ iff for all $d \in D$, $\models^- A_x(c_d)$.

where there is a corresponding constant c_d such that $\delta(c_d) = d$, for every $d \in D$ in the language.

For interpreting the indefinite descriptions, we also need to introduce another semantic device Φ into structure. An interpretation $\mathcal{S}$ for **PMn** is a triple $\langle D, \delta, \Phi\rangle$, in which $\langle D, \delta\rangle$ is defined as above, and

$$\Phi : D \times D \to D$$

is a choice function, s.t. for $\langle X, Y\rangle \in D \times D$, $\Phi(\langle X, Y\rangle) \in X$ if $X \neq \varnothing$, otherwise, $\Phi(\langle X, Y\rangle) = d$ for arbitary $d \in D$. Then the denotation of ε-terms is:

- $\delta(\varepsilon xA) = \Phi(\langle\{d : \models^+ A_x(c_d)\}, \{d : \models^- A_x(c_d)\}\rangle)$

Definition 3 (Semantic consequence relation)**.** Let Σ be a set of formulas, the sematic consequence relation $\Sigma \models A$ is defined in terms of truth-preserving, that is, for every interpretation, if $\models^+ B$ for all $B \in \Sigma$, then $\models^+ A$.

3.3 Concrete Interpretation for PMn

In this subsection, we will give a concrete interpretation which makes every axioms and rules in **PMn** true. It means that **PMn** is not vacuous system. The paradox of nothing is solved in the sense that it is both true and false in this interpretation.

Definition 4 (Concrete Interpretation)**.** Let $\mathcal{S} = (D, \delta, \Phi)$ be an interpretation for **PMn**, which is defined as follows:

- $D = \{\top, a, b, \bot\}$.
- $\delta(\mathbf{n}) = \bot$, $\delta(\mathbf{e}) = \top$, $\delta(\mathbf{c_a}) = a$ and $\delta(\mathbf{c_b}) = b$.
- $\delta(<) = (\delta^+(<), \delta^-(<))$ s.t. $\delta^+(<) = \{(a, \top), (b, \top), (\bot, \top)\}$ and $\delta^-(<) = \{(\top, \top), (\top, a), (\top, b), (\top, \bot), (a, a), (a, b), (a, \bot), (b, a), (b, b), (b, \bot), (\bot, \top), (\bot, a), (\bot, b), (\bot, \bot)\}$.

- $\delta(=) = (\delta^+(=), \delta^-(=))$ s.t. $\delta^+(=) = \{(\top,\top),(a,a),(b,b),(\bot,\bot)\}$ and $\delta^-(<) = \{(\top,a),(\top,b),(\top,\bot),(a,\top),(a,b),(a,\bot),(b,\top),(b,a),(b,\bot),$ $(\bot,\top),(\bot,a),(\bot,b),(\bot,\bot)\}$.
- Φ is an arbitary choice function defined as above.

We use the following tables to illustrate the structure above:

$<$	$\top$	a	b	$\bot$
$\top$	$-$	$-$	$-$	$-$
a	$+$	$-$	$-$	$-$
b	$+$	$-$	$-$	$-$
$\bot$	$\pm$	$-$	$-$	$-$

$=$	$\top$	a	b	$\bot$
$\top$	$+$	$-$	$-$	$-$
a	$-$	$+$	$-$	$-$
b	$-$	$-$	$+$	$-$
$\bot$	$-$	$-$	$-$	$\pm$

where $+$ indicates the membership of the extension (only), $-$, membership of the anti-extension (only), and $\pm$ indicates both. By the definition of $\leq$, $\circ$, and $\bullet$, the interpretation for them are illustrated as below:

$\leq$	$\top$	a	b	$\bot$
$\top$	$+$	$-$	$-$	$-$
a	$+$	$+$	$-$	$-$
b	$+$	$-$	$+$	$-$
$\bot$	$\pm$	$-$	$-$	$\pm$

$\circ$	$\top$	a	b	$\bot$
$\top$	$+$	$+$	$+$	$\pm$
a	$+$	$+$	$-$	$-$
b	$+$	$-$	$+$	$-$
$\bot$	$\pm$	$-$	$-$	$\pm$

$\bullet$	$\top$	a	b	$\bot$
$\top$	$-$	$-$	$-$	$\pm$
a	$-$	$-$	$+$	$+$
b	$-$	$+$	$-$	$+$
$\bot$	$\pm$	$+$	$+$	$\pm$

In what follows, we are going to check all characteristic axioms are true in this interpretation, other axioms and rules can be checked in the standard way.

Proposition 1 (Soundness)**. M1, M2, M2+, Mc3, Mc3+, M4n, M4n+, M4e, M4e+, M5** *are true in* $\mathcal{S}$.

Proof. Given $\mathcal{S} = (D, \delta, \Phi)$ defined as in Definition 4.

M1 $x < y \wedge y < z \to x < z$[7]

It is vacuously true, since it can never be the case that both conjuncts of the antecedent are true ($+$ or $\pm$). Hence the antecedent is always $-$.

M2 $x < y \to \neg y < x$

We list all combinations below in which the antecedent is $+$ or $\pm$. It is not difficult to see the consequent is always true in all cases. Hence **M2** is true in $\mathcal{S}$.

[7]Notice that the verification of a conditional requires to establish the truth-preservation forward and falsity-preservation backwards, that is, if $A \to B$ is true, it is required that:

- if A is $+$ or $\pm$, then B is $+$ or $\pm$;
- if B is $-$ or $\pm$, then A is $-$ or $\pm$, or equivalently, if A is $+$, then B is $+$.

x	y	$x < y$	$\neg(y < x)$
a	$\top$	$+$	$+$
b	$\top$	$+$	$+$
$\bot$	$\top$	$\pm$	$+$

M2+ $\mathfrak{A}z(z \circ x \leftrightarrow z \circ y) \rightarrow x = y$

It suffices to prove if the consequent is $-$, then the antecedent is $-$; and if the consequent is $\pm$, then the antecedent is $\pm$. Ignoring symmetries, we list all values of x, y which makes the consequent $-$ or $\pm$; and the third column is the value of z to make the antecedent $-$ or $\pm$. The first four lines requires z to make the antecedent $-$, while the fifth line requires z to make the antecedent $\pm$.

x	y	
$\top$	a	b
$\top$	$\bot$	a (or b)
a	b	a (or b)
a	$\bot$	a
$\bot$	$\bot$	$\top$ (or $\bot$)

Mc3 $\mathfrak{A}v\mathfrak{S}x(v \bullet x \wedge \mathfrak{A}y(y \neq y \vee y \circ x \vee y \circ v))$

We firstly list all the possible values of v in the first column of the following table, and give a witness v' for x; the corresponding values of the first and second conjuncts are in the third and fourth columns. And we can check that every value of y other than $\bot$ makes the disjunction $y \neq y \vee y \circ x \vee y \circ v$ to be $+$, and $\bot$ makes it $\pm$:

v	v'		
$\top$	$\bot$	$\pm$	$\pm$
a	b	$+$	$\pm$
b	a	$+$	$\pm$
$\bot$	$\top$	$\pm$	$\pm$

Hence **Mc3** takes the value $\pm$.

Moreover, ignoring the symmetry between a and b, in the following table, the first and second columns contains all the the values of v and all possible values of x other than v':

v	x	
$\top$	$\top$	
$\top$	a	
a	$\top$	
a	a	b
a	$\bot$	b
$\bot$	a	b
$\bot$	$\bot$	a or b

The value of v, x in the first three lines makes the first conjunct of **M3c** $-$. Meanwhile, in the last four lines there is a value of y which makes the second conjunct $-$. Hence in all cases the conjunction is $-$. So v' is the only value of x which makes $Comp(v, x)$ true.

Mc3+ $Comp(v, x_1) \wedge Comp(v, x_2) \rightarrow x_1 = x_2$

It suffices to prove if the consequent is $-$, then the antecedent is $-$; and if the consequent is $\pm$, then the antecedent is $\pm$ or $-$. That is, if $x_1 = x_2$ is $-$, then as is discussed above, at least one of the conjuncts is $-$, hence the conjunction is $-$; if $x_1 = x_2$ is $\pm$, then $x_1 = x_2 = \bot$, hence the conjunction is $\pm$(when v is $\top$) or $-$(otherwise). Hence, for any value of v, the set of things which satisfy $Comp(v, x)$ is a singleton, $\{v'\}$. So any Y, $\Phi(\langle\{v'\}, Y\rangle) = v'$; and the value of $\varepsilon x Comp(v, x)$, that is, $\bar{v}$, is v'.

M4n $\mathfrak{S}z\mathfrak{A}y(y \circ z \leftrightarrow \mathfrak{S}x(y \circ x \wedge x \neq x))$

$\bot$ is a witness of z, for consider:

– $y \circ \bot \leftrightarrow \mathfrak{S}x(x \neq x \wedge y \circ x)$

In the following table, the left column is the four values of y, and the values of the corresponding left and right sides of the biconditional are in the middle and right columns:

$\top$	$\pm$	$\pm$
a	$-$	$-$
b	$-$	$-$
$\bot$	$\pm$	$\pm$

The middle column can be calculated immediately. In the right column, when y is $\top$ or $\bot$, take x to be $\bot$ will make the conjunction $\pm$, and when y is a or b, the conjunction is $-$.

Moreover, $\bot$ is the only witness, since **M2+** holds in the structure. That is, the set of objects in the domain which witness z is $\{\bot\}$. By semantics of Φ, for any Y, $\Phi\langle\{\bot\}, Y\rangle = \bot$, and so the denotation of **n**, that is $\varepsilon z\mathfrak{A}y(y \circ z \leftrightarrow \mathfrak{S}x(y \circ x \wedge x \neq x))$, is $\bot$.

M4n+ $\mathfrak{A}x(x \neq x \rightarrow x \leq \mathbf{n})$

Only $\bot$ makes $x \neq x$ true, and the value of $\bot \leq \bot$ is $\pm$, so truth is preserved forward. $x \neq x$ is always false in this interpretation. So falsity is preserved backwards.

M4e $\mathfrak{S}z\mathfrak{A}y(y \circ z \leftrightarrow \mathfrak{S}x(y \circ x \land x = x))$
$\top$ is a witness for z. For consider:

- $y \circ \top \leftrightarrow \mathfrak{S}x(y \circ x \land x = x)$

In the following table, the left column is the four values of y, and the values of the corresponding left and right sides of the biconditional are in the middle and right columns:

$\top$	$+$	$+$
a	$+$	$+$
b	$+$	$+$
$\bot$	$\pm$	$\pm$

For the last column: In the first three lines, give x the value in the first column in the same line. In the last line, both $\top$ and $\bot$ make the conjunction $\pm$ (the other values giving $-$), which makes the $\mathfrak{S}$ to be $\pm$.
Moreover, $\top$ is the only witness, since **M2+** holds in the structure. That is, the set of objects in the domain which witness z is $\{\top\}$. By semantics of Φ, for any Y, $\Phi\langle\{\top\}, Y\rangle = \top$, and so the denotation of **e**, that is $\varepsilon z\mathfrak{A}y(y \circ z \leftrightarrow \mathfrak{S}x(y \circ x \land x = x))$, is $\top$.

M4e+ $\mathfrak{A}x(x = x \rightarrow x \leq \mathbf{e})$
We can check in the beginning table that $x \leq \top$ is always true($+$ or $\pm$) in this interpretation, i.e. truth is preserved forward. The only value which makes $x \leq \top$ false is $\bot$, and $\bot = \bot$ is false, hence falsity is preserved backward.

M5 $\mathfrak{S}xA \rightarrow A_x(\varepsilon xA)$
If $\models^+ \mathfrak{S}xA$, then there is some c s.t. $A_x(c)$ is $+$, hence $\{d :\models^+ A_x(c_d)\}$ is non-empty, then $\delta(\varepsilon xA) \in \{d :\models^+ A_x(c_d)\}$, w.l.o.g., let it be $\delta(c_0)$, i.e. εxA is c_0, then $\models^+ A_x(c_0)$. So the truth is preserved forward.
If $\models^- A_x(\varepsilon xA)$, it means that $\delta(\varepsilon xA) \notin \{d :\models^+ A_x(c_d)\}$. By semantics of Φ we know that $\{d :\models^+ A_x(c_d)\}$ is $\varnothing$, i.e. A has no extension, hence $\models^- \mathfrak{S}xA$. So the falsity is preserved backward.

□

4 A Paracomplete Solution to Paradox of Nothing

A logic is paracomplete if there are some formulas such that these formulas and their negations are not true simutaneously, that is, the excluded law doesn't hold in this logic. In this section, we are going to propose another approach, which is based on a paracomplete logic, to deal with the paradox of nothing. We will show that even though we accept Priest's definition for **nothing**, there still has another way to solve the paradox. Based on Łukasiewicz's three valued logic $\mathbf{L}_3$, we construct a new axiomatic system **L3Mn** for theory of nothing , which is a $\mathbf{L}_3$ based mereology system. We will give a concrete model for this theory and show that both "**nothing** is something" and "**nothing** is no thing" are neither true nor false in this model, and hence the paradox can be solved.

On the one hand, we agree with Priest on that nothing could refer to an object as a noun and we could use some characterizations to pick it out. On the other hand, like Quine's famous criticism of the existence of possible objects ([6, pp.3–5]), we could use the similar argument to argue that we cannot even claim that $x = x$ when x is not an existent object for being lack of the criteria of identity. Therefore, the relation between **everything** and **nothing** is not symmetric. As the mereological fusion of all existent objects, **everything** is an existent object when we accept the principle of general composition, but as the mereological fusion of both all existent and non-existent objects, it is not an existent object anymore. While **nothing**, as the mereological fusion of no things, cannot be an existent object. Therefore, when we talk about **nothing**, it is always an non-existent object but it is not the case for **everything**. In this sense, when we define **everything** as the object which is the mereological fusion of every thing that is self-identical, we always means the existent **everything** since we cannot claim that $x = x$ when x is not an existent object.

4.1 Axiomatic system L3Mn

In the language, we make distinctions between the standard quantifiers ($\exists, \forall$), which quantify over existent objects, and general quantifiers ($\mathfrak{S}, \mathfrak{A}$), which quantify over both existent and non-existent objects in the language. Correspondingly, we also introduce two classes of constants, one for existent objects and the other for both existent and non-existent objects. We use $e_1, \ldots, e_n, \ldots$ to stand for the former and $c_1, \ldots, c_n, \ldots$ to stand for the latter in the language [8]. The axiomatic system **L3Mn** contains the following axioms and rules:

1. Mereology axioms:

M1 $x < y \wedge y < z \rightarrow x < z$

M2 $x < y \rightarrow \neg y < x$

M2r $\neg x < x$

M2+' $\forall x \forall y \mathfrak{A} z (z \circ x \leftrightarrow z \circ y) \rightarrow x = y$

M3c' $\mathfrak{A} v \mathfrak{S} x (v \bullet x \wedge \forall y (y \circ x \vee y \circ v))$

[8] For the objects nothing and everything, we use **n** and **e** instead of e_n and c_e respectively.

M3c+' $Comp(v, x_1) \wedge Comp(v, x_2) \rightarrow v = \mathbf{e} \vee x_1 = x_2$
M4n $\mathfrak{S}z\mathfrak{A}y(y \circ z \leftrightarrow \mathfrak{S}x(y \circ x \wedge x \neq x))$
M4e' $\mathfrak{S}z\mathfrak{A}y(y \circ z \leftrightarrow \exists x(y \circ x \wedge x = x))$
M4n+ $\mathfrak{A}x(x \neq x \rightarrow x \leq \mathbf{n})$
M4e+' $\forall x(x = x \rightarrow x \leq \mathbf{e})$
M5 $\mathfrak{S}xA \rightarrow A_x(\varepsilon xA)$
M6 $\mathbf{n} \bullet \mathbf{e}$

2. Paracomplete first-order logic axioms :

I1 $A \rightarrow (B \rightarrow A)$
I2 $(A \rightarrow B) \rightarrow ((B \rightarrow C) \rightarrow (A \rightarrow C))$
I3 $(A \rightarrow (B \rightarrow C)) \rightarrow (B \rightarrow (A \rightarrow C))$
I4 $((A \rightarrow B) \rightarrow B) \rightarrow ((B \rightarrow A) \rightarrow A)$
I5 $((((A \rightarrow B) \rightarrow A) \rightarrow A) \rightarrow (B \rightarrow C)) \rightarrow (B \rightarrow C)$
C1 $A \wedge B \rightarrow A$
C2 $A \wedge B \rightarrow B$
C3 $(A \rightarrow B) \rightarrow ((A \rightarrow C) \rightarrow (A \rightarrow B \wedge C))$
D1 $A \rightarrow A \vee B$
D2 $B \rightarrow A \vee B$
D3 $(A \rightarrow C) \rightarrow ((B \rightarrow C) \rightarrow (A \vee B \rightarrow C))$
N1 $(\neg B \rightarrow \neg A) \rightarrow (A \rightarrow B)$
P12 $\mathfrak{A}xA \rightarrow A_x(c)$
P12' $\forall xA \rightarrow A_x(e)$
P13 $\forall x x = x$
P14 $\forall x x \leq \mathbf{e}$

3. Rules:

MP $A, A \rightarrow B \vdash B$
Adj $A, B \vdash A \wedge B$
Aff $A \rightarrow B, C \rightarrow D \vdash (B \rightarrow C) \rightarrow (A \rightarrow D)$
UG $A \vdash \mathfrak{A}xA$
QE $\mathfrak{A}xA \vdash \forall xA$
Subst $a = b \vdash A_x(a) \leftrightarrow A_x(b)$

For the reasons discussed above, the axioms **M3c, M3c+, M4e, M4e+** in **PMn** is replaced with **M3c',M3c+', M4e', M4e+'** respectively, moreover, we modify the definition of $Comp(u, v)$ as below:

$$Comp(u, v) ::= \ u \bullet v \wedge \forall y(y \circ x \vee y \circ v)$$

From the view of proof theory, the paradox of nothing, symbolized as

$$\mathfrak{S}xx = \mathbf{n} \wedge \neg\mathfrak{S}xx = \mathbf{n}$$

is not provable in **L3Mn**. However, using **M3c'**, we can still prove $Comp(\mathbf{e}, \mathbf{n})$ as below, it means **nothing** is the absence of everything intuitively.

Proof.

1.	$\mathbf{n} \bullet \mathbf{e}$	M6
2.	$\forall x x \leq \mathbf{e}$	M4e+', P13, MP
3.	$x \circ \mathbf{e}$	definition of $\circ$
4.	$(x \circ \mathbf{e}) \vee (x \circ \mathbf{n})$	3, D1, MP
5.	$\forall x((x \circ \mathbf{e}) \vee (x \circ \mathbf{n}))$	4, UG'
6.	$\mathbf{e} \bullet \mathbf{n} \wedge \forall x((x \circ \mathbf{e}) \vee (x \circ \mathbf{n}))$	1, 5, Adj

□

Moreover, $\forall x \neg x \leq \mathbf{n}$ is an immediate consequence of the axiom **P15** as below:

Proof.

1.	$\mathfrak{A}x(\neg x \leq \mathbf{n} \vee \neg x \leq \mathbf{e})$	definition of $\mathfrak{A}$, $\bullet$, De Morgan
2.	$\forall x(\neg x \leq \mathbf{n} \vee \neg x \leq \mathbf{e})$	1, QE
3.	$\forall x x \leq \mathbf{e}$	P14
4.	$\forall x \neg x \leq \mathbf{n}$	2, 3, DS

□

4.2 Semantics for L3Mn

In the following sections, we are going to show that the axiomatic system **L3Mn** is not trivial, it defines some models. In this subsection, we introduce the interpretation for **L3Mn** and give a concrete model in the next subsection.

Definition 5 (Structure). An interpretation for **L3Mn** without ε-terms is a tuple $\mathcal{S} = \langle D, D^{\varnothing}, \delta \rangle$ which satisfies:

1. $\mathbf{n} \in \mathbf{D}^{\varnothing}$ and $\mathbf{e} \in D$;
2. $\delta(c) \in D$ for any existent constant c, and $\delta(e) \in D^{\varnothing}$ for any non-existent constant e;
3. $\delta(P) = (\delta^{+}(P), \delta^{-}(P)) \subseteq (D \cup D^{\varnothing})^n \times (D \cup D^{\varnothing})^n$, s.t. $\delta^{+}(P) \cap \delta^{-}(P) = \varnothing$ for any n-ary predicate P;
4. $\delta(=) = (\delta^{+}(=), \delta^{-}(=)) = (\{(d, d) \mid d \in D\}, \{(d_1, d_2) \mid d_1, d_2 \in D \cup D^{\varnothing}$ and $d_1 \neq d_2\})$.

Clearly, besides true and false, there is also a third-value, neither true nor false, in the structure, we use i to denote this value. Let $\models^{+}$, $\models^{-}$ and $\models^{i}$ stand for truth, falsity and truth-value gap with respect to a structure $\mathcal{S}$ respectively, then the truth conditions for the language is defined inductively as follows: let P be an n-place predicate:

- $\models^+ P(c_1, \ldots, c_n)$ iff $\langle \delta(c_1), \ldots, \delta(c_n) \rangle \in \delta^+(P)$.
- $\models^- P(c_1, \ldots, c_n)$ iff $\langle \delta(c_1), \ldots, \delta(c_n) \rangle \in \delta^-(P)$.
- $\models^i P(c_1, \ldots, c_n)$, otherwise.

- $\models^+ \neg A$ iff $\models^- A$.
- $\models^- \neg A$ iff $\models^+ A$.
- $\models^i \neg A$, otherwise.

- $\models^+ A \wedge B$ iff $\models^+ A$ and $\models^+ B$.
- $\models^- A \wedge B$ iff $\models^- A$ or $\models^- B$.
- $\models^i A \wedge B$, otherwise.

- $\models^+ A \vee B$ iff $\models^+ A$ or $\models^+ B$.
- $\models^- A \vee B$ iff $\models^- A$ and $\models^- B$.
- $\models^i A \vee B$, otherwise.

- $\models^+ A \rightarrow B$ iff (1) $\models^- A$, or (2) $\models^+ B$, or (3) $\models^i A$ and $\models^i B$.
- $\models^- A \rightarrow B$ iff $\models^+ A$ and $\models^- B$.
- $\models^i A \rightarrow B$, otherwise.

For the quantifiers, we augment the language with a constant, c_d for every $d \in D$ such that $\delta(c_d) = d$, and d for any element in D or $D^{\varnothing}$ without making distinctions in syntax and semantics. $A_x(c)$ is A with every free occurrence of x replaced by c, and $A_x(d)$ is defined respectively.

- $\models^+ \mathfrak{A}xA$ iff for all $d \in D \cup D^{\varnothing}$, $\models^+ A_x(d)$.
- $\models^- \mathfrak{A}xA$ iff for some $d \in D \cup D^{\varnothing}$, $\models^- A_x(d)$.
- $\models^i \mathfrak{A}xA$, otherwise.

- $\models^+ \forall xA$ iff for all $d \in D$, $\models^+ A_x(c_d)$.
- $\models^- \forall xA$ iff for some $d \in D$, $\models^- A_x(c_d)$.
- $\models^i \forall xA$, otherwise.

- $\models^+ \mathfrak{S}xA$ iff for some $d \in D \cup D^{\varnothing}$, $\models^+ A_x(d)$.
- $\models^- \mathfrak{S}xA$ iff for all $d \in D \cup D^{\varnothing}$, $\models^- A_x(d)$.
- $\models^i \mathfrak{S}xA$, otherwise.

- $\models^+ \exists xA$ iff for some $d \in D$, $\models^+ A_x(c_d)$.

- $\models^- \exists xA$ iff for all $d \in D$, $\models^- A_x(c_d)$.
- $\models^i \exists xA$, otherwise.

Like the interpretation for **PMn**, an interpretation $\mathcal{S}$ for **L3Mn** is a quadruple $\langle D, D^{\varnothing}, \delta, \Phi\rangle$, in which $\langle D, D^{\varnothing}, \delta\rangle$ is defined as above, and

$$\Phi : (D \cup D^{\varnothing})^3 \to D \cup D^{\varnothing}$$

is a choice function, s.t. $\Phi(\langle X, Y, Z\rangle) \in X$ if $X \neq \varnothing$; and $\Phi(\langle X, Y, Z\rangle) \in Y$ if $X = \varnothing$ and $Y \neq \varnothing$; otherwise, $\Phi(\langle X, Y, Z\rangle) = \mathbf{n}$, for any $\langle X, Y, Z\rangle \in (D \cup D^{\varnothing})^3$. Then the denotation of ε-terms is:

- $\delta(\varepsilon xA) = \Phi(\langle\{d : \models^+ A_x(c_d)\}, \{d : \models^i A_x(c_d)\}, \{d : \models^- A_x(c_d)\}\rangle)$

Definition 6 (Semantic consequence relation)**.** Let Σ be a set of formulas, the sematic consequence relation $\Sigma \models A$ is defined in terms of truth-preserving, that is, for every interpretation, if $\models^+ B$ for all $B \in \Sigma$, then $\models^+ A$.

Proposition 2. $Comp(\mathbf{e}, \mathbf{n})$ *is true in* **L3Mn**.

Proof. We first show that for any $c \neq \mathbf{n}$, $\models^- c \circ \mathbf{n}$. That is because for any $d \in D \cup D^{\varnothing}$, if $d \in D$, then $\models^- c_d \leq \mathbf{n}$; if $d \in D^{\varnothing}$, then $\models^- c_d \leq c$. Hence $\models^- \mathbf{e} \circ \mathbf{n}$, $\models^+ \mathbf{e} \bullet \mathbf{n}$.
For any $d \in D$, $\models^+ c_d \leq \mathbf{e}$, hence $\models^+ \forall y(y \circ \mathbf{e} \vee y \circ \mathbf{n})$, finally we get $\models^+ Comp(\mathbf{e}, \mathbf{n})$. □

In our semantics, to say **nothing** is or is not something is neither true or false.

Proposition 3. $\models^i \mathfrak{S}xx = \mathbf{n}$*, and* $\models^i \neg\mathfrak{S}xx = \mathbf{n}$.

Proof. For any $d \in D \cup D^{\varnothing}$, if $d \in D$, then by interpretation of $=$, $\models^- c_d = \mathbf{n}$; if $d \in D$, then by interpretation of $=$, $\models^i c_d = \mathbf{n}$, notice that in this case c_d is exactly $\mathbf{n}$. Hence we get $\models^i \mathfrak{S}xx = \mathbf{n}$. By semantics of $\neg$, we can easily get $\models^i \neg\mathfrak{S}xx = \mathbf{n}$. □

4.3 Concrete Interpretation for L3Mn

In this subsection, we will give a concrete interpretation which makes every axioms and rules in **L3Mn** true. It means that **L3Mn** is not vacuous system. The paradox of nothing is solved in the sense that it is neither true nor false in this interpretation.

Definition 7 (Concrete Interpretation)**.** Let $\mathcal{S} = (D, D^{\varnothing}, \delta, \Phi)$ be an interpretation for **L3Mn**, which is defined as follows:

- $D = \{\top, a, b\}$, $D^{\varnothing} = \{\bot\}$.
- $\delta(\mathbf{n}) = \bot$, $\delta(\mathbf{e}) = \top$, $\delta(\mathbf{c_a}) = a$, and $\delta(\mathbf{c_b}) = b$.

- $\delta(<) = (\delta^+(<), \delta^-(<))$ s.t. $\delta^+(<) = \{(a, \top), (b, \top)\}$, and $\delta^-(<) = \{(\top, \top), (\top, a), (\top, b), (\top, \bot), (a, a), (a, b), (a, \bot), (b, a), (b, b), (b, \bot), (\bot, \top), (\bot, a), (\bot, b), (\bot, \bot)\}$.
- $\delta(=) = (\delta^+(=), \delta^-(=))$ s.t. $\delta^+(=) = \{(\top, \top), (a, a), (b, b)\}$ and $\delta^-(<) = \{(\top, a), (\top, b), (\top, \bot), (a, \top), (a, b), (a, \bot), (b, \top), (b, a), (b, \bot), (\bot, \top), (\bot, a), (\bot, b)\}$.
- Φ is an arbitary choice function defined as above.

We use the following tables to illustrate the structure above:

$<$	$\top$	a	b	$\bot$
$\top$	$-$	$-$	$-$	$-$
a	$+$	$-$	$-$	$-$
b	$+$	$-$	$-$	$-$
$\bot$	$-$	$-$	$-$	$-$

$=$	$\top$	a	b	$\bot$
$\top$	$+$	$-$	$-$	$-$
a	$-$	$+$	$-$	$-$
b	$-$	$-$	$+$	$-$
$\bot$	$-$	$-$	$-$	i

where $+$ indicates the membership of the extension (only), $-$, membership of the anti-extension(only), and $\pm$ indicates both. By the definition of $\leq$, $\circ$, and $\bullet$, the interpretation for them are illustrated as below:

$\leq$	$\top$	a	b	$\bot$
$\top$	$+$	$-$	$-$	$-$
a	$+$	$+$	$-$	$-$
b	$+$	$-$	$+$	$-$
$\bot$	$-$	$-$	$-$	i

$\circ$	$\top$	a	b	$\bot$
$\top$	$+$	$+$	$+$	$-$
a	$+$	$+$	$-$	$-$
b	$+$	$-$	$+$	$-$
$\bot$	$-$	$-$	$-$	i

$\bullet$	$\top$	a	b	$\bot$
$\top$	$-$	$-$	$-$	$+$
a	$-$	$-$	$+$	$+$
b	$-$	$+$	$-$	$+$
$\bot$	$+$	$+$	$+$	i

In what follows, we are going to check all characteristic axioms are true in this interpretation, other axioms and rules can be checked in the standard way.

Proposition 4 (Soundness)**. M1, M2, M2r, M2+, Mc3', Mc3+', M4n, M4n+, M4e', M4e+', M5, M6** *are true in* $\mathcal{S}$.

Given $\mathcal{S} = (D, D^{\varnothing}, \delta, \Phi)$ defined as in Definition 7.

M1 $x < y \wedge y < z \rightarrow x < z$

It is vacuously true, since it can never be the case that both conjuncts of the antecedent are $+$ (or i). Hence the antecedent is always $-$.

M2 $x < y \rightarrow \neg y < x$

We list all combinations below in which the antecedent is $+$. It is not difficult to see the consequent is always $+$ when the antecedent is $+$ in all cases. Hence **M2** is true in $\mathcal{S}$.

x	y	$x < y$	$\neg(y < x)$
a	$\top$	$+$	$+$
b	$\top$	$+$	$+$

M2r $\neg x < x$

It can be easily check in the table of the interpretation of $<$.

M2+' $\forall x \mathfrak{A} y(\mathfrak{A} z(z \circ x \leftrightarrow z \circ y) \to x = y)$

It suffices to show that if the consequent is $-$ (or i), so is the antecedent. Considering there are standard quantifiers in the beginning, there is no such pair of x, y making the consequent i. Omitting the symmetric cases, we list all pairs of x, y who makes the consequent $-$, and show the witness of the falsity of the antecedent:

x	y	
a	$\top$	b
b	$\top$	a
a	b	a (or b)
a	$\bot$	a
b	$\bot$	b

Mc3' $\mathfrak{A} v \mathfrak{S} x(v \bullet x \wedge \forall y(y \circ x \vee y \circ v))$

The first column in the table below contains all the possible values of v. The second is a witness, v', for x, and the third and fourth are the corresponding values of the first and second conjuncts, the truth is guaranteed by the interpretation of $\circ$ and $\bullet$.

v	v'		
$\top$	$\bot$	$+$	$+$
a	b	$+$	$+$
b	a	$+$	$+$
$\bot$	$\top$	$+$	$+$

Hence **Mc3'** is true.

Moreover, ignoring the symmetry between a and b, in the following table, the first column contains all the the values of v. The second column contains all possible values of x other than v':

v	x	
$\top$	$\top$	
$\top$	a	
a	$\top$	
$\bot$	$\bot$	
a	a	b
a	$\bot$	b
$\bot$	a	b

In the first four lines, the value of the first conjunct of **Mc3'** is $-$ or i. In the last three lines there is a value of y which makes the second conjunct $-$, which is shown in the third column. Hence in all cases the conjunction is $-$. So v' is the only value of x which makes $Comp(v, x)$ true.

Mc3+' $Comp(v, x_1) \wedge Comp(v, x_2) \rightarrow v = \mathbf{e} \vee x_1 = x_2$

We have just seen that for any value of v, unless the interpretations of x_1 and x_2 are both v', the value of the antecedent is $-$. If they are, the value of the antecedent is $+$, and the value of the consequent is $+$; in particular, if v is **e**, then the left disjunct is $+$ but the right one is i.

As **Mc3+'** shows, for any value of v, the set of things which satisfy $Comp(v, x)$ is a singleton, $\{v'\}$. Hence, for any Y, Z, $\Phi\langle\{v'\}, Y, Z\rangle = v'$; and the value of $\varepsilon x Comp(v, x)$, that is, $\bar{v}$, is v'.

M4n $\mathfrak{S}z\mathfrak{A}y(y \circ z \leftrightarrow \mathfrak{S}x(y \circ x \wedge x \neq x))$

$\bot$ is a witness of z, for consider:

– $y \circ \bot \leftrightarrow \mathfrak{S}x(x \neq x \wedge y \circ x)$

In the following table, the left column is the four values of y, and the values of the corresponding left and right sides of the biconditional are in the middle and right columns:

$\top$	$-$	$-$
a	$-$	$-$
b	$-$	$-$
$\bot$	i	i

The only x who makes $x \neq x$ not $-$ is $\bot$, however unless $y = \bot$, $y \circ \bot$ is $-$, hence in the first three cases rightside of the biconditional is $-$, and in the last case it is i:

Moreover, $\bot$ is the only witness, for any other interpretation of z, just take y as $\bot$, then the left side of the biconditional is always $-$, and meanwhile $\bot$ is a witness of the right side being i. That is, the set of objects in the domain which witness z is $\{\bot\}$. Hence, for any Y, Z, $\Phi\langle\{\bot\}, Y, Z\rangle = \bot$, and so the denotation of **n**, that is $\varepsilon z\mathfrak{A}y(y \circ z \leftrightarrow \mathfrak{S}x(y \circ x \wedge x \neq x))$, is $\bot$.

M4n+ $\mathfrak{A}x(x \neq x \rightarrow x \leq \mathbf{n})$

There is no value makes $x \neq x$ true, and the only value makes $x \neq x$ in the value of i, is $\bot$. But the value of $\bot \leq \bot$ is also i. So **M4n+** is valid.

M4e' $\mathfrak{S}z\mathfrak{A}y(y \circ z \leftrightarrow \exists x(y \circ x \wedge x = x))$

$\top$ is a witness for z. For consider:

– $y \circ \top \leftrightarrow \exists x(y \circ x \wedge x = x)$

In the following table, the left column is the four values of y, and the values of the corresponding left and right sides of the biconditional are in the middle and right columns:

$\top$	$+$	$+$
a	$+$	$+$
b	$+$	$+$
$\bot$	$-$	$-$

For the last column: In the first three lines, the value of x in the first column gives the result. In the last line, for there is no such $x \in D$ makes $\bot \circ x$ true or i, hence the conjunction is $-$.

Moreover, $\top$ is the only witness. For any other values of z, as shown in the following table. In the first two lines, the value of y makes the left side of the biconditional $-$, and the value of x is a witness of the right side; in the last line, the value of y makes the left side i, and there is no witness of the right side, hence the right side is $-$.

z	y	x
a	b	b
b	a	a
$\bot$	$\bot$	

That is, the set of objects in the domain which witness z is $\{\top\}$. Hence, for any Y, Z, $\Phi\langle\{\top\}, Y, Z\rangle = \top$, and so the denotation of **e**, that is $\varepsilon z \mathfrak{A} y(y \circ z \leftrightarrow \mathfrak{S} x(y \circ x \wedge x = x))$, is $\top$.

M4e+' $\forall x(x = x \to x \leq \mathbf{e})$

It can be checked in the table at the beginning of subsection 4.3.

M5 $\mathfrak{S} x A \to A_x(\varepsilon x A)$

If $\mathfrak{S} x A$ is $+$, then there is some c, s.t. $A_x(c)$ is $+$, and for $\delta(\varepsilon x A) = \Phi(\langle\{\delta(c) : \models^+ A_x(c)\}, \{d : \models^i A_x(c_d)\}, \{d : \models^- A_x(c_d)\}\rangle)$, w.l.o.g., let $\delta(\varepsilon x A) = c_0$, by semantics of Φ, for $\{\delta(c) : \models^+ A_x(c)\}$ is non-empty, hence $c_0 \in \{\delta(c) : \models^+ A_x(c)\}$, i.e. $\models^+ A_x(\varepsilon x A)$.

If $\mathfrak{S} x A$ is i, then $\{d : \models^+ A_x(c_d)\}$ is empty but $\{d : \models^i A_x(c_d)\}$ is non-empty, hence by semantics of Φ, $\delta(\varepsilon x A) \in \{d : \models^i A_x(c_d)\}$, i.e. $\models^i A_x(\varepsilon x A)$.

In summary, **M5** is true in both of these two cases.

M6 **n** $\bullet$ **e**

It can be checked in the table at the beginning of 7.

From semantic theoretic perspective, the paradox of nothing, $\mathfrak{S} x x = \mathbf{n} \wedge \neg \mathfrak{S} x x = \mathbf{n}$ is neither true nor false in **LMn3**. But it doesn't mean that every proposition involving **nothing** is neither true nor false, there are still some true propositions for **nothing**, e.g., it is the absence of everything.

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“无”之悖论：一个弗完全解决方案

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摘　要

“无”的概念包含着一个深刻的哲学谜题，它既是又不是一个东西，因而形成了一个悖论。为了解决这一悖论，普里斯特提出了一种基于弗协调逻辑和分体论的刻画“无”的理论。本文则提出了另一种解决该悖论的方法，即基于弗完全（卢卡西维茨三值逻辑）和分体论的形式系统。本文将论证，即便我们接受普里斯特对于“无”的刻画，我们也没有必要接受关于“无”的矛盾命题。

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