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The Completeness for the Combination of PDL and EL with Perfect Recall and No Miracles*

Yanjun Li

Abstract. This paper proves the completeness for the combination of propositional dynamic logic and single-agent epistemic logic in which the modalities interact. The kinds of interactions we consider are two commuting axioms, namely, the axiom of perfect recall and the axiom of no miracles. These two axioms capture the interactions between actions and knowledge.

1 Introduction

Propositional dynamic logic (PDL) is an important logic for reasoning about programs or actions. ([4]) Epistemic logic (EL) is a modal logic concerned with reasoning about informational aspects of agent, in particular, agent's belief and knowledge. ([7]) Thus the combination of EL and PDL is a powerful tool for reasoning about interactions between knowledge and actions.

There are two ways to combine PDL and EL: product and fusion. ([5]) In this paper, we combine PDL and EL by way of fusion. It is shown that the fusion of two modal logics inherit properties such as completeness, the finite model property and decidability from the individual logics. ([8, 9, 17]) However, it is much more complex if the fusion is extended with interactions of these two logics.

The most frequently discussed interactions in the combinations of PDL and EL are *perfect recall*, *no learning* and *Church-Rosser* axiom. The conjunction of perfect recall and no learning is also called *commutativity* in [5]. Perfect recall (PR) is commonly formulated by the axiom schema

$$\langle a\rangle\hat{\mathcal{K}}\phi \to \hat{K}\langle a\rangle\phi$$

where $\langle a\rangle$ is a PDL modality and $\hat{\mathcal{K}}$ is the epistemic modality. It expresses the persistence of the agent's knowledge after the execution of an action. No learning (NL) is given by the axiom schema

$$\hat{\mathcal{K}}\langle a\rangle\phi \to \langle a\rangle\hat{\mathcal{K}}\phi.$$

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Yanjun Li College of Philosophy, Nankai University
lyjlogic@nankai.edu.cn

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The formula says the agent knows the result of his action in advance. In other words, there is no learning. Church-Rosser axiom (CR) is the axiom schema

$$\langle a\rangle\mathcal{K}\phi \to \mathcal{K}\langle a\rangle\phi.$$

It says that if an agent is possible to know ϕ by performing an action a then he knows that it is possible to achieve ϕ by doing a.

The fusions of PDL and EL extended with various choices of PR, NL and CR are well studied in [5, 12, 14, 13]. For example, it has been shown that the fusion of PDL and EL extended with PR and NL coincides with the product of PDL and EL. It has also been proved that the fusion of PDL and EL extended with NL is the same as the fusion of PDL and EL extended with CR, which is consistent with the fact that NL and CR are equal to each other in EL models.

Besides PR, NL and CR, *no miracles* is also an important property which captures the interaction between agent's actions and knowledge. ([2, 15, 1, 16, 10, 11]) No miracles (NM) is syntactically given by the axiom schema

$$\hat{\mathcal{K}}\langle a\rangle\phi \to [a]\hat{\mathcal{K}}\phi.$$

Please note that the structure of the NM axiom differs from the above NL axiom in the form of the outer modality in the succedent: in NM it is a box modality $[a]$ while in NL it is a diamond modality $\langle a\rangle$. This is because not all actions are executable at the current world. This subtle difference makes NM more suitable to capture the interaction between knowledge and actions and NL more suitable to describe the interaction between knowledge and time. A more detailed discussion of the difference between NM and NL can be found in [15]. Intuitively, no miracles expresses that there is no "miracles" situation such that the agent cannot distinguish two states initially but nevertheless he can distinguish the states resulting from executing the same action on these two states.

While the combinations of PDL and EL extended with PR, NL or CR are well studied, the combination extended with NM and these properties has not been investigated sufficiently. In this paper, we prove the completeness for the combination of PDL $\oplus$ EL extended with PR and NM. Please note that even though the completeness for PDL $\oplus$ EL $\oplus$ $\{\mathrm{PR}, \mathrm{NL}\}$ is proved in [13], the completeness for PDL $\oplus$ EL $\oplus$ $\{\mathrm{PR}, \mathrm{NM}\}$ is not a simple copy of its method. NM makes a big difference. The difficulty lies in the fact that the filtration cannot preserve the no-miracles property of models. Therefore, we have to construct a proper model based on the filtration model step by step. This will be more detailly illustrated in the paper.

The paper is organized as follows. Section 2 introduces the language, model and semantics of PDL $\oplus$ EL, and the properties that correspond to the axioms PR and NM. Section 3 presents the filtration method. Section 4 constructs a model with the desired properties from the filtration model. Section 5 proves the completeness of PDL $\oplus$ EL $\oplus$ $\{\mathrm{PR}, \mathrm{NM}\}$, and we conclude in Section 6.

2 Preliminaries

It is assumed that the reader has some basic familiarity with PDL and standard modal logics. A good reference on modal logics is [3] and more background on PDL can be found in [6].

Let A be a countable set of atomic actions, and P be a countable set of proposition letters.

Definition 1 (Language) The *Language* $\mathcal{L}$ we consider is an extension of PDL ([6]) with the epistemic modality $\mathcal{K}$ of EL, which is as follows:

$$\phi ::= p \mid \neg\phi \mid (\phi \wedge \phi) \mid [\pi]\phi \mid \mathcal{K}\phi$$
$$\pi ::= a \mid ?\phi \mid (\pi;\pi) \mid (\pi + \pi) \mid \pi^*$$

where $p \in \mathrm{P}$, $a \in \mathrm{A}$.

We will often omit parentheses when doing so ought not cause confusion. As usual, we use the following abbreviations: $\phi \vee \psi := \neg(\neg\phi \wedge \neg\psi)$, $\phi \to \psi := \neg\phi \vee \psi$, $\langle\pi\rangle\phi := \neg[\pi]\neg\phi$, $\hat{\mathcal{K}}\phi := \neg\mathcal{K}\neg\phi$.

Definition 2 (Model) A standard model $\mathcal{M}$ is a quadruple $\langle W, R, \{Q(a) \mid a \in \mathrm{A}\}, V\rangle$ where W is a non-empty set of states, $Q(a)$ is a binary relation on W, R is an equivalence relation on W and $V : \mathrm{P} \to \mathcal{P}(W)$ is an assignment function. A pointed standard model is a pair $(\mathcal{M}, s)$ consisting of a standard model $\mathcal{M}$ and a state $s \in W$.

Given a standard model $\mathcal{M}$, we also write $(s,t) \in Q(a)$ as $s \xrightarrow{a} t$, write $(s,t) \in R$ as $s \sim t$, and write the set $\{u \mid s \sim u\}$ as $[s]$. Moreover, we use $[s] \xrightarrow{a} t$ to denote that there are $s' \in [s]$ and $t' \in [t]$ such that $s' \xrightarrow{a} t'$. Moreover, Q can be extended as a function on all actions by the following rules:

$$\begin{aligned}
Q(\pi_1 + \pi_2) &\stackrel{\text{def}}{=} Q(\pi_1) \cup Q(\pi_2) \\
Q(\pi_1;\pi_2) &\stackrel{\text{def}}{=} Q(\pi_1) \circ Q(\pi_2) \\
Q(\pi^*) &\stackrel{\text{def}}{=} \textstyle\bigcup_{n\in\omega} Q(\pi^n) \\
Q(?\phi) &\stackrel{\text{def}}{=} \{(s,s) \mid \mathcal{M}, s \vDash \phi\}
\end{aligned}$$

where $\mathcal{M}, s \vDash \phi$ is defined as follows.

Definition 3 (Semantics) Given a formula $\phi \in \mathcal{L}$ and a pointed standard model $(\mathcal{M}, s)$, the satisfaction relation $\vDash$ is defined as follows.

$$\begin{array}{lcl}
\mathcal{M}, s \vDash p & \iff & s \in V(p) \\
\mathcal{M}, s \vDash \neg\phi & \iff & \mathcal{M}, s \nvDash \phi \\
\mathcal{M}, s \vDash \phi \wedge \psi & \iff & \mathcal{M}, s \vDash \phi \text{ and } \mathcal{M}, s \vDash \psi \\
\mathcal{M}, s \vDash \mathcal{K}\phi & \iff & (s,u) \in R \text{ implies } \mathcal{M}, u \vDash \phi \\
\mathcal{M}, s \vDash [\pi]\phi & \iff & (s,t) \in Q(\pi) \text{ implies } \mathcal{M}, t \vDash \phi
\end{array}$$

A *logic* in $\mathcal{L}$ is a set of $\mathcal{L}$-formulas that contains all propositional tautologies and is closed under *uniform substitution* and the following rules:

$$\phi, \phi \to \psi \vdash \psi \qquad\qquad \phi \vdash [\pi]\phi \qquad\qquad \phi \vdash \mathcal{K}\phi$$

Let Γ and Δ be any subsets of $\mathcal{L}$-formulas. By $\Gamma \oplus \Delta$ we denote the least logic which contains both Γ and Δ. The combination of propositional dynamic logic and epistemic logic is therefore denoted by $\mathrm{PDL} \oplus \mathrm{EL}$[1].

A *non-standard* model for $\mathcal{L}$ is a quadruple $\langle W, R, Q, V\rangle$ that satisfies all the properties of a standard model except that, in a non-standard model, there is $Q(\pi^*) \supseteq \bigcup_{n\in\omega} \pi^n$, but it still validates all the formulas of the logic $\mathrm{PDL}\oplus\mathrm{EL}$. In the following text, if it does not matter whether the model is standard or non-standard, we will just call it model.

Definition 4 (Properties of com^{pr} and com^{nm}) A model $\mathcal{M}$ has the property of

- Perfect recall (com^{pr}) if for all $a \in \mathrm{A}$ and all $s, t, t' \in W$, $(s,t) \in Q(a)$ and $(t,t') \in R$ imply that there exists $s' \in W$ such that $(s,s') \in R$ and $(s',t') \in Q(a)$.
- No miracles (com^{nm}) if for all $a \in \mathrm{A}$ and all $s, s', t, t' \in W$, $(s,t) \in Q(a)$, $(s,s') \in R$ and $(s',t') \in Q(a)$ imply $(t,t') \in R$.

The extension of $\mathrm{PDL} \oplus \mathrm{EL}$ with PR and NM is denoted by $\mathrm{PDL} \oplus \mathrm{EL} \oplus \{\mathrm{PR}, \mathrm{NM}\}$. We leave the soundness for the logic $\mathrm{PDL} \oplus \mathrm{EL} \oplus \{\mathrm{PR}, \mathrm{NM}\}$ w.r.t. models with $com^{pr,nm}$ to the reader; in this paper, we will focus on the completeness for the logic $\mathrm{PDL} \oplus \mathrm{EL} \oplus \{\mathrm{PR}, \mathrm{NM}\}$ w.r.t. models with $com^{pr,nm}$.

3 Filtration $\mathcal{M}^\Sigma$

To show the completeness of an extension of PDL, the common method is to do filtration on models through a finite *Fischer-Ladner* closure.

Definition 5 (Fischer-Ladner closure) A *Fischer-Ladner* closure Σ is the minimal set of formulas satisfying:

- if $\psi \in \Sigma$ then $\neg\psi \in \Sigma$, provided ψ does not start with $\neg$;
- if $\neg\psi$, $K\psi$ or $[\pi]\psi$ are in Σ then $\chi \in \Sigma$;
- if $\psi \wedge \chi \in \Sigma$ then $\psi, \chi \in \Sigma$;
- if $[\pi_1;\pi_2]\psi \in \Sigma$ then $[\pi_1][\pi_2]\psi \in \Sigma$;
- if $[\pi_1 + \pi_2]\psi \in \Sigma$ then both $[\pi_1]\psi$ and $[\pi_2]\psi$ are in Σ;
- if $[?\chi]\psi \in \Sigma$ then $\chi \in \Sigma$;

[1]Here PDL represents the set of axioms of propositional dynamic logic, and EL represents the set of axioms of epistemic logic.

- if $[\pi^*]\psi \in \Sigma$ then $[\pi][\pi^*]\psi \in \Sigma$.

In this paper, we only consider *Fischer-Ladner* closure that is finite. So, in the following context, we always assume that the *Fischer-Ladner* closure Σ is finite. Moreover, we need the following notations to introduce filtration.

$$\begin{array}{rcl} \Sigma(s) & \stackrel{\text{def}}{=} & \{\phi \in \Sigma \mid \mathcal{M}, s \vDash \phi\} \\ \Sigma([s]) & \stackrel{\text{def}}{=} & \{\Sigma(t) \mid t \in [s]\} \end{array}$$

Please note that $\Sigma(u)$ is a finite set of formulas since we assume that Σ is finite. Therefore, we also see $\Sigma(u)$ as a formula which is $\bigwedge_{\phi \in \Sigma(u)} \phi$.

Let $\mathcal{M}$ be a model and Σ be a finite *Fischer-Ladner* closure. The filtration of $\mathcal{M}$ through Σ is as follows.

Definition 6 (Filtration $\mathcal{M}^\Sigma$) The relation $\equiv_\Sigma$ is an equivalence relation on the domain of $\mathcal{M}$, which is defined as follows.

$$s \equiv_\Sigma u \iff \Sigma(s) = \Sigma(u), \text{ and } \Sigma([s]) = \Sigma([u])$$

We denote the equivalence class of a state s with respect to the equivalence relation $\equiv_\Sigma$ by $|s|$. The filtration $\mathcal{M}^\Sigma = \langle W^\Sigma, R^\Sigma, \{Q^\Sigma(a) \mid a \in \mathrm{A}\}, V^\Sigma\rangle$ is defined as follows

$$\begin{array}{rcl} W^\Sigma & \stackrel{\text{def}}{=} & \{|s| \mid s \in W\} \\ (|s|, |u|) \in R^\Sigma & \iff & \text{there are } s' \in |s|, u' \in |u| : (s', u') \in R \\ (|s|, |t|) \in Q^\Sigma(a) & \iff & \text{there are } s' \in |s|, t' \in |t| : (s', t') \in Q(a) \\ V^\Sigma(p) & \stackrel{\text{def}}{=} & \{|s| \mid s \in V(p)\}, \text{ for all proposition letter } p \in \Sigma \end{array}$$

The equivalence relation $\equiv_\Sigma$ and the filtration model $\mathcal{M}^\Sigma$ above are the same as in [13]. It is also shown in [13] that each equivalence class $|s|$ is defined by the following formula:

$$\phi_{[s]} \stackrel{\text{def}}{=} (\bigwedge_{u \in [s]} \hat{\mathcal{K}}\Sigma(u)) \wedge \mathcal{K}(\bigvee_{u \in [s]} \Sigma(u)).$$

Proposition 7 Given a model $\mathcal{M}$, states $s, u \in W$, and a finite *Fischer-Ladner* closure Σ, we have that $s \equiv_\Sigma u$ if and only if $\mathcal{M}, u \vDash \Sigma(s) \wedge \phi_{[s]}$.

With the proposition above, by a standard process shown in [6], we have the following lemma.

Lemma 1 (Filtration Lemma) Let $\mathcal{M}$ be a model and Σ be a finite *Fischer-Ladner* closure.

(i) For each $\phi \in \Sigma$, $\mathcal{M}, s \vDash \phi \iff \mathcal{M}^\Sigma, |s| \vDash \phi$

(ii) For each $\langle\pi\rangle\phi \in \Sigma$, if $(|s|, |t|) \in Q^{\Sigma}(\pi)$ and $\mathcal{M}, t \vDash \phi$ then $\mathcal{M}, s \vDash \langle\pi\rangle\phi$.

The filtration $\mathcal{M}^{\Sigma}$ can preserve the properties of models: perfect recall, no learning, and Church-Rosser property, since the filtration $\mathcal{M}^{\Sigma}$ has the following property.

Proposition 8 If $(|s|, |u|) \in R^{\Sigma}$, we have that for each $s' \in |s|$ there exists $u' \in |u|$ such that $(s', u') \in R$, and that for each $u' \in |u|$ there exists $s' \in |s|$ such that $(s', u') \in R$.

Proof Let $(s, u) \in R$. Given $s' \in |s|$, we will show that there is some $u' \in |u|$ such that $(s', u') \in R$. Since $s' \in |s|$, this follows that $\Sigma([s']) = \Sigma([s])$. Since $(s, u) \in R$, this follows that there is some $u' \in [s']$ such that $\Sigma(u') = \Sigma(u)$. Moreover, since $\Sigma([u']) = \Sigma([s'])$ and $\Sigma([s]) = \Sigma([u])$, this follows that $\Sigma([u']) = \Sigma([u])$. Thus, we have $u' \equiv_{\Sigma} u$, namely, $u' \in |u|$. Since R^{Σ} is an equivalence relation, from a similar process we can show that for each $u' \in |u|$ there exists $s' \in |s|$ such that $(s', u') \in R$.

□

The following proposition follows from Proposition 8 immediately.

Proposition 9 If $\mathcal{M}$ has the property of com^{pr}, so does $\mathcal{M}^{\Sigma}$.

However, Proposition 8 cannot guarantee that the property of com^{nm} is preserved after filtration. Therefore, we will need to construct a new model that has the property com^{nm} based on $\mathcal{M}^{\Sigma}$. We found that $\mathcal{M}^{\Sigma}$ has the property stated in the following proposition. This property will play an important role and helps us to construct a model with $com^{pr,nm}$ based on $\mathcal{M}^{\Sigma}$.

Proposition 10 Given $\mathcal{M}$ with com^{nm}, $(|s|, |t|) \in Q^{\Sigma}(a)$, $(|s|, |u|) \in R^{\Sigma}$, and $\langle a\rangle\phi \in \Sigma$, if $\mathcal{M}^{\Sigma}, |u| \vDash \langle a\rangle\phi$, then there exists v such that $(|t|, |v|) \in R^{\Sigma}$, $(|u|, |v|) \in Q^{\Sigma}(a)$, and $\mathcal{M}^{\Sigma}, |v| \vDash \phi$.

Proof Since $(|s|, |t|) \in Q^{\Sigma}(a)$, let $(s, t) \in Q(a)$. Since $(|s|, |u|) \in R^{\Sigma}$, it follows by Proposition 8 that there is some $u' \in |u|$ such that $(s, u') \in R$. Since $\mathcal{M}^{\Sigma}, |u| \vDash \langle a\rangle\phi$ and $\langle a\rangle\phi \in \Sigma$, it follows by Lemma 1 that $\mathcal{M}, u' \vDash \langle a\rangle\phi$. Therefore, there is some v such that $\mathcal{M}, v \vDash \phi$ and $(u', v) \in Q(a)$. Thus we have $(|u|, |v|) \in Q^{\Sigma}(a)$, and $\mathcal{M}^{\Sigma}, |v| \vDash \phi$. Since $\{(s, t), (u', v)\} \subseteq Q(a)$, $(s, u') \in R$, and $\mathcal{M}$ has the property of com^{nm}, this follows that $(t, v) \in R$. By the definition of $\mathcal{M}^{\Sigma}$, we have that $(|t|, |v|) \in R^{\Sigma}$. □

4 Step by step

Due to Proposition 10, this section will use the step-by-step method ([3]) to construct a model with $com^{pr,nm}$ from $\mathcal{M}^{\Sigma}$ where $\mathcal{M}$ has the properties com^{pr} and

com^{nm}. For simplicity's sake, rather than clarify it every time, we assume that all models $\mathcal{M}$ discussed in this section have the properties com^{pr} and com^{nm}.

Definition 11 (Network $\mathcal{N}$) A network $\mathcal{N}$ of $\mathcal{M}^{\Sigma}$ is a quadruple $\mathcal{N} = \langle N, \sim, \{\xrightarrow{a} \mid a \in \mathrm{A}\}, L\rangle$ where N is a non-empty set of states, $\sim$ is an equivalence relation on N, $\xrightarrow{a}$ is a binary relation on N, and L is a labeling function mapping each state in N to a state in $\mathcal{M}^{\Sigma}$.

For each action π, $\xrightarrow{\pi}$ is also a binary relation on N, which is defined as

$$\begin{array}{lcl}
\xrightarrow{?\psi} & \stackrel{\text{def}}{=} & \{(x,x) \mid \mathcal{M}^{\Sigma}, L(x) \vDash \psi\} \\
\xrightarrow{\pi_1+\pi_2} & \stackrel{\text{def}}{=} & \xrightarrow{\pi_1} \cup \xrightarrow{\pi_2} \\
\xrightarrow{\pi_1;\pi_2} & \stackrel{\text{def}}{=} & \xrightarrow{\pi_1} \circ \xrightarrow{\pi_2} \\
\xrightarrow{\pi^*} & \stackrel{\text{def}}{=} & \bigcup_n \xrightarrow{\pi^n}
\end{array}$$

Definition 12 (Coherent) A network $\mathcal{N}$ of $\mathcal{M}^{\Sigma}$ is *coherent* if it satisfies:

(C1) If $(|s|, L(x)) \in R^{\Sigma}$ then there exists $y \in N$ such that $L(y) = |s|$ and $x \sim y$;
(C2) $\mathcal{N}$ is a deterministic tree on the level of belief states, i.e. $[x] \xrightarrow{a} [y]$ and $[x] \xrightarrow{a} [y']$ imply that $[y] = [y']$ for each $a \in \mathrm{A}$;
(C3) If $x \sim y$ then $(L(x), L(y)) \in R^{\Sigma}$;
(C4) For each $a \in \mathrm{A}$, if $x \xrightarrow{a} y$ then $(L(x), L(y)) \in Q^{\Sigma}(a)$.

Definition 13 (Saturated) A network $\mathcal{N}$ of $\mathcal{M}^{\Sigma}$ is *saturated* if it satisfies:

(S1) If $y' \sim y$ and $[x] \xrightarrow{a} [y]$ where $a \in \mathrm{A}$, then there is some $x' \in [x]$ such that $x' \xrightarrow{a} y'$, namely, $\mathcal{N}$ has the property com^{pr};
(S2) For each $\langle\pi\rangle\phi \in \Sigma$, if $\mathcal{M}^{\Sigma}, L(x) \vDash \langle\pi\rangle\phi$ for some $x \in N$, then there is some $y \in N$ such that $x \xrightarrow{\pi} y$ and $\mathcal{M}^{\Sigma}, L(y) \vDash \phi$.

Definition 14 (Defect) Let $\mathcal{N}$ be a network of $\mathcal{M}^{\Sigma}$. An *S1-defect* consists of a triple $([x], y', a)$ for which there is $[x] \xrightarrow{a} [y']$ while y' has no a-predecessor in $[x]$. An *S2-defect* of $\mathcal{N}$ consists of a node $x \in N$ and a formula $\langle\pi\rangle\phi \in \Sigma$ for which $\mathcal{M}^{\Sigma}, L(x) \vDash \langle\pi\rangle\phi$, and there is no $y \in N$ such that $x \xrightarrow{\pi} y$ and $\mathcal{M}^{\Sigma}, L(y) \vDash \phi$.

Definition 15 (Extension) Let $\mathcal{N}' = \langle N', \sim', \{\xrightarrow{a}{}' \mid a \in \mathrm{A}\}, L'\rangle$ and $\mathcal{N} = \langle N, \sim, \{\xrightarrow{a} \mid a \in \mathrm{A}\}, L\rangle$ be two networks. We say that $\mathcal{N}'$ *extends* $\mathcal{N}$ (notation: $\mathcal{N}' \rhd \mathcal{N}$) if $\langle N, \sim, \{\xrightarrow{a} \mid a \in \mathrm{A}\}\rangle$ is a subframe of $\langle N', \sim', \{\xrightarrow{a}{}' \mid a \in \mathrm{A}\}\rangle$ and L' agrees L on N.

Lemma 2 (Repair Lemma) Let $\mathcal{N} = \langle N, \sim, \{\xrightarrow{a} \mid a \in \mathrm{A}\}, L\rangle$ be a finite, coherent network of $\mathcal{M}^{\Sigma}$. For any defect of $\mathcal{N}$, there is a finite, coherent $\mathcal{N}' \rhd \mathcal{N}$ lacking this defect.

Proof Let $\mathcal{N} = \langle N, \sim, \{\xrightarrow{a} \mid a \in \mathrm{A}\}, L\rangle$ be a finite, coherent network of $\mathcal{M}^{\Sigma}$, We prove the lemma by showing that both types of defect can be removed.

S1-defects:
Let $([x], y', a)$ where $a \in \mathrm{A}$ be an S1-defect of $\mathcal{N}$. Assume that $x \xrightarrow{a} y$ for some $y \in [y']$. It follows by (C3) and (C4) that $(L(x), L(y)) \in Q^{\Sigma}(a)$ and $(L(y), L(y')) \in R^{\Sigma}$. It follows by Proposition 9 that there is some $|u|$ such that $(|u|, L(x)) \in R^{\Sigma}$ and $(|u|, L(y')) \in Q^{\Sigma}(a)$. By (C1), we have that there is some x' such that $L(x') = |u|$ and $x' \sim x$. Let $\mathcal{N}' = \langle N', \sim', \{\xrightarrow{a}{}' \mid a \in \mathrm{A}\}, L'\rangle$ be the same as $\mathcal{N}$ except that $\xrightarrow{a}{}' \stackrel{\text{def}}{=} \xrightarrow{a} \cup \{(x', y')\}$. This follows that $\mathcal{N}' \rhd \mathcal{N}$.

It is obvious that $\mathcal{N}'$ satisfies (C1), (C3) and (C4). Next we will check (C2). If $x'' \xrightarrow{a}{}' y''$ and $x'' \sim' x'$, we need to show that $y'' \sim' y'$. Firstly we have that $x'' \sim' x$ since $\sim'$ is an equivalence relation. Since $\mathcal{N}$ satisfies (C2), this follows that $y'' \sim' y$. Since $y \sim' y'$, this follows that $y'' \sim' y'$.

$$\begin{array}{ccccc} x & \sim' & x' & \sim' & x'' \\ \downarrow a & & \downarrow a & & \downarrow a \\ y & \sim' & y' & \sim' & y'' \end{array}$$

S2-defects:
Let $(x, \langle\pi\rangle\phi)$ be an S2-defect of $\mathcal{N}$. This follows that $\langle\pi\rangle\phi \in \Sigma$, and $\mathcal{M}^{\Sigma}, L(x) \vDash \langle\pi\rangle\phi$. We will show that there is a finite, coherent network $\mathcal{N}' \rhd \mathcal{N}$ in which this defect is removed. We prove it by induction on π.

1. For an atomic action $a \in \mathrm{A}$, there are two cases: (i) there are states $x', y' \in N$ such that $x' \xrightarrow{a} y'$ and $x \sim x'$; (ii) there are no such states x', y'. For (i), since $\mathcal{N}$ is coherent, this follows by (C3) and (C4) that $(L(x'), L(y')) \in Q^{\Sigma}(a)$ and $(L(x), L(x')) \in R^{\Sigma}$. Since $\mathcal{M}^{\Sigma}, L(x) \vDash \langle a\rangle\phi$, it follows by Proposition 10 that there is some $|v|$ such that $(|v|, L(y')) \in R^{\Sigma}$, $(L(x), |v|) \in Q^{\Sigma}(a)$ and $\mathcal{M}^{\Sigma}, |v| \vDash \phi$. It follows by (C1) that there is some y such that $L(y) = |v|$ and $y' \sim y$. Let $\mathcal{N}' = \langle N', \sim', \{\xrightarrow{a}{}' \mid a \in \mathrm{A}\}, L'\rangle$ be the same as $\mathcal{N}$ except that $\xrightarrow{a}{}' \stackrel{\text{def}}{=} \xrightarrow{a} \cup \{(x, y)\}$. This follows that $\mathcal{N}' \rhd \mathcal{N}$. It is obvious that $\mathcal{N}'$ satisfies (C1), (C3) and (C4). By a similar process of S1-defects above, it can be shown that $\mathcal{N}'$ also satisfies (C2).

   For (ii), it follows by $\mathcal{M}^{\Sigma}, L(x) \vDash \langle a\rangle\phi$ that there is some $|t|$ such that $\mathcal{M}^{\Sigma}, |t| \vDash \phi$ and $(L(x), |t|) \in Q^{\Sigma}(a)$. Let $D = \{|v| \mid (|v|, |t|) \in R^{\Sigma}\}$. Since $\mathcal{M}^{\Sigma}$ is a finite model, this follows that D is finite. Let D' be a set of *new* nodes (new in the sense that $z \notin N$ for each $z \in D'$), and f be a one-to-one mapping from D' to D. Let $y \in D'$ and $f(y) = |t|$. Hence, we define $\mathcal{N}' = \langle N', \sim', \{\xrightarrow{a}{}' \mid a \in

$\mathrm{A}\}, L'\rangle$ as follows

$$\begin{aligned}
&N' = N \cup D' \\
&\sim' = \sim \cup \{(z, z') \mid z, z' \in D'\} \\
&\xrightarrow{a}{}' = \xrightarrow{a} \cup \{(x, y)\} \\
&L' = L \cup f.
\end{aligned}$$

It is obvious that $\mathcal{N}'$ is finite. Next we will show that $\mathcal{N}'$ is coherent. It is easy to check that $\mathcal{N}'$ satisfies (C1), (C3) and (C4). To check (C2), we only need to show that if $x' \xrightarrow{a}{}' y'$ and $x \sim' x'$ then $y \sim' y'$. By the assumption of (ii), we know that there are no such x', y' in $\mathcal{N}$. This follows that $y = y'$. By the definition of $\sim'$ above, we have $y \sim' y$, namely $y \sim' y'$.

2. For a test $?\psi$, it is impossible. In other words, there is no defect in the form of $(x, \langle ?\psi\rangle\phi)$. The reason is the following: if $\mathcal{M}^\Sigma, L(x) \vDash \langle ?\psi\rangle\phi$, this follows that $\mathcal{M}^\Sigma, L(x) \vDash \psi$. Then, we have that $x \xrightarrow{?\psi} x$.
3. For the case $\pi_1 + \pi_2$, firstly we have that $\mathcal{M}^\Sigma, L(x) \vDash \langle \pi_i\rangle\phi$ for some $i = 1$ or $i = 2$. Since $(x, \langle \pi_1 + \pi_2\rangle\phi)$ is an S2-defect of $\mathcal{N}$, this follows that $(x, \langle \pi_i\rangle\phi)$ is an S2-defect of $\mathcal{N}$. By induction on π, this follows that there is a finite, coherent network $\mathcal{N}' \rhd \mathcal{N}$ in which the defect $(x, \langle \pi_i\rangle\phi)$ is removed. Thus, the defect $(x, \langle \pi_1 + \pi_2\rangle\phi)$ is also removed in $\mathcal{N}'$.
4. For the case $\pi_1; \pi_2$, firstly we have that $\mathcal{M}^\Sigma, L(x) \vDash \langle \pi_1\rangle\langle \pi_2\rangle\phi$. If the pair $(x, \langle \pi_1\rangle\langle \pi_2\rangle\phi)$ is a defect of $\mathcal{N}$, it follows by IH that there is a finite, coherent network $\mathcal{N}' \rhd \mathcal{N}$ in which the defect $(x, \langle \pi_1\rangle\langle \pi_2\rangle\phi)$ is removed. Thus, there is some $y \in N'$ such that $x \xrightarrow{\pi_1} y$ and $\mathcal{M}^\Sigma, L(y) \vDash \langle \pi_2\rangle\phi$. If $(y, \langle \pi_2\rangle\phi)$ is still a defect of $\mathcal{N}'$, it follows by IH that there a finite, coherent network $\mathcal{N}'' \rhd \mathcal{N}'$ in which the defect $(x, \langle \pi_2\rangle\phi)$ is removed. This follows that $\mathcal{N}'' \rhd \mathcal{N}$ and the defect $(x, \langle \pi_1\rangle\langle \pi_2\rangle\phi)$ is removed in $\mathcal{N}''$.
5. For the case π^*, firstly we have that $\mathcal{M}^\Sigma, L(x) \vDash \neg\phi \wedge \langle \pi\rangle\langle \pi^*\rangle\phi$ since $(x, \langle \pi^*\rangle\phi)$ is a defect. We assume that $(x, \langle \pi\rangle\langle \pi^*\rangle\phi)$ is also a defect. By IH, this follows that there is a finite, coherent extension of $\mathcal{N}$ in which this defect is removed. By (C2), we assume that $[x]$ is a leaf-node of $\mathcal{N}$. Since $\mathcal{M}^\Sigma, L(x) \vDash \langle \pi^*\rangle\phi$, this follows that $\mathcal{M}^\Sigma, L(x) \vDash \langle \pi^n\rangle\phi$ for some natural number n, and we assume that n is the minimal natural number satisfying the condition. Thus, there are $|s|_1, \cdots, |s|_n$ such that $L(x) \xrightarrow{\pi} |s|_1, \cdots, \xrightarrow{\pi} |s|_n$, $\mathcal{M}^\Sigma, |s|_i \vDash \langle \pi\rangle\langle \pi^*\rangle\phi$ where $1 \leq i < n$, and $\mathcal{M}^\Sigma, |s|_n \vDash \phi$. Then, by IH, there are $\mathcal{N}_n \rhd \cdots \rhd \mathcal{N}_1 \rhd \mathcal{N}$ such that each $\mathcal{N}_i$ where $1 \leq i \leq n$ is a finite, coherent network. Since $[x]$ is a leaf-node of $\mathcal{N}$, we can make that x_i is a new node to $\mathcal{N}_{i-1}$, and that $x_{i-1} \xrightarrow{\pi} x_i$, and that $L^i(x_i) = |s|_i$, where $1 < i \leq n$. Therefore, in $\mathcal{N}_n$, we have that $x \xrightarrow{\pi^*} x_n$ and $\mathcal{M}^\Sigma, L^n(x_n) \vDash \phi$, that is, the defect $(x, \langle \pi^*\rangle\phi)$ is removed in $\mathcal{N}_n$.

$\square$

Definition 16 (Model $\mathfrak{I}_{\mathcal{N}}$) Let $\mathcal{N} = \langle N, \sim, \{\xrightarrow{a} \mid a \in \mathrm{A}\}, L\rangle$ be a network of $\mathcal{M}^{\Sigma}$. The model $\mathfrak{I}_{\mathcal{N}}$ is defined as $\langle N, R, \{Q(a) \mid a \in \mathrm{A}\}, V\rangle$ where $R \stackrel{\text{def}}{=} \sim$, $Q(a) \stackrel{\text{def}}{=} \xrightarrow{a}$, and V is defined by $V(p) = \{x \mid \mathcal{M}^{\Sigma}, L(x) \vDash p\}$.

Lemma 3 (Truth Lemma for $\mathfrak{I}_{\mathcal{N}}$) Let $\mathcal{N}$ be a coherent, saturated network of $\mathcal{M}^{\Sigma}$. We have the following results:

(i) For each $\phi \in \Sigma$, $\mathfrak{I}_{\mathcal{N}}, x \vDash \phi \iff \mathcal{M}^{\Sigma}, L(x) \vDash \phi$;
(ii) For each $\langle\pi\rangle\phi \in \Sigma$, if $x \xrightarrow{\pi} y$ in $\mathcal{N}$ then $(x, y) \in Q(\pi)$ in $\mathfrak{I}_{\mathcal{N}}$;
(iii) For each $\langle\pi\rangle\phi \in \Sigma$, if $(x, y) \in Q(\pi)$ then $(L(x), L(y)) \in Q^{\Sigma}(\pi)$.

Proof We prove the lemma by simultaneous induction on (i), (ii) and (iii). We start with (i). There are five cases, depending on the form of ϕ. We will only focus on the cases of $\hat{\mathcal{K}}\phi$ and $\langle\pi\rangle\phi$; the other cases are straightforward.

- For $\hat{\mathcal{K}}\phi$, if $\mathfrak{I}_{\mathcal{N}}, x \vDash \hat{\mathcal{K}}\phi$ then there is some $y \in N$ such that $(x, y) \in R$ and $\mathfrak{I}_{\mathcal{N}}, y \vDash \phi$. By the definition of $\mathfrak{I}_{\mathcal{N}}$, we have $x \sim y$ in $\mathcal{N}$. Since $\mathcal{N}$ is coherent, it follows by (C3) that $(L(x), L(y)) \in R^{\Sigma}$. By IH, we have that $\mathcal{M}^{\Sigma}, L(y) \vDash \phi$. Thus, we have $\mathcal{M}^{\Sigma}, L(x) \vDash \hat{\mathcal{K}}\phi$.
  If $\mathcal{M}^{\Sigma}, L(x) \vDash \hat{\mathcal{K}}\phi$, then there is some $|u|$ such that $(L(x), |u|) \in R^{\Sigma}$ and $\mathcal{M}^{\Sigma}, |u| \vDash \phi$. Since $\mathcal{N}$ is coherent, it follows by (C1) that there is some $y \in N$ such that $L(y) = |u|$ and $x \sim y$. It follows that $\mathfrak{I}_{\mathcal{N}}, y \vDash \phi$ and $(x, y) \in R$. Thus, we have $\mathfrak{I}_{\mathcal{N}}, x \vDash \hat{\mathcal{K}}\phi$.
- For $\langle\pi\rangle\phi$, if $\mathfrak{I}_{\mathcal{N}}, x \vDash \langle\pi\rangle\phi$ then there is some $y \in N$ such that $(x, y) \in Q(\pi)$ and $\mathfrak{I}_{\mathcal{N}}, y \vDash \phi$. By (iii), we have that $(L(x), L(y)) \in Q^{\Sigma}(\pi)$. By IH, we have that $\mathcal{M}^{\Sigma}, L(y) \vDash \phi$. Thus, we have $\mathcal{M}^{\Sigma}, L(x) \vDash \langle\pi\rangle\phi$.
  If $\mathcal{M}^{\Sigma}, L(x) \vDash \langle\pi\rangle\phi$, since $\mathcal{N}$ is saturated, it follows by (S2) that there is some $y \in N$ such that $x \xrightarrow{\pi} y$ in $\mathcal{N}$ and $\mathcal{M}^{\Sigma}, L(y) \vDash \phi$. It follows by IH that $\mathfrak{I}_{\mathcal{N}}, y \vDash \phi$. Since $x \xrightarrow{\pi} y$ in $\mathcal{N}$, it follows by (ii) that $(x, y) \in Q(\pi)$ in $\mathfrak{I}_{\mathcal{N}}$. Thus, we have $\mathfrak{I}_{\mathcal{N}}, x \vDash \langle\pi\rangle\phi$.

For (ii), we will only focus on the case that π is a test $?\psi$; the other cases are straightforward by the definition of $\mathfrak{I}_{\mathcal{N}}$ and by IH. If $x \xrightarrow{?\psi} x$ in $\mathcal{N}$, this follows taht $\mathcal{M}^{\Sigma}, L(x) \vDash \psi$. It follows by IH that $\mathfrak{I}_{\mathcal{N}}, x \vDash \psi$. Thus, we have $(x, x) \in Q(?\psi)$ in $\mathfrak{I}_{\mathcal{N}}$.

For (iii), we will only focus on the cases that π is an atomic action a, or a test $?\psi$, or a Kleene star π^*; the other cases are straightforward by IH.

- For an atomic action a, if $(x, y) \in Q(a)$ in $\mathfrak{I}_{\mathcal{N}}$, it follows by the definition that $x \xrightarrow{a} y$ in $\mathcal{N}$. Since $\mathcal{N}$ is coherent, it follows by (C4) that $(L(x), L(y)) \in Q^{\Sigma}(a)$.
- For a test $?\psi$, if $(x, y) \in Q(?\psi)$ in $\mathfrak{I}_{\mathcal{N}}$, this follows that $x = y$ and $\mathfrak{I}_{\mathcal{N}}, x \vDash \psi$. By IH, this follows that $\mathcal{M}^{\Sigma}, L(x) \vDash \psi$. Thus, we have that $(L(x), L(x)) \in$

$Q^\Sigma(?\psi)$.

- For a Kleene star π^*, if $(x, y) \in Q(\pi^*)$ in $\mathfrak{I}_\mathcal{N}$, this follows that there are a natural number n and some states $x_0 \cdots x_n \in N$ such that $x_0 = x$, $x_n = y$, and $x_i \xrightarrow{\pi} x_{i+1}$ for all $0 \leq i < n$. It follows by IH that $(L(x_i), L(x_{i+1})) \in Q^\Sigma(\pi)$ for all $0 \leq i < n$. Thus, we have $(L(x), L(y)) \in Q^\Sigma(\pi^*)$.

□

Proposition 17 Let $\mathcal{N}$ be a coherent, saturated network of $\mathcal{M}^\Sigma$. The model $\mathfrak{I}_\mathcal{N}$ has the properties of $com^{pr,nm}$.

Proof Since the frame of $\mathfrak{I}_\mathcal{N}$ is the same as the frame of $\mathcal{N}$, we then only need to show that $\mathcal{N}$ has the properties of $com^{pr,nm}$. Since $\mathcal{N}$ is coherent, it follows by (C2) that $\mathcal{N}$ has the property com^{nm}. Since $\mathcal{N}$ also is saturated, it follows by (S1) that $\mathcal{N}$ has the property com^{pr}. □

5 Completeness

This section will show the completeness of PDL$\oplus$EL$\oplus\{$PR, NM$\}$ with respect to models with com^{pr} and com^{nm}.

Definition 18 (Canonical model $\mathfrak{M}$) The canonical model for PDL$\oplus$EL$\oplus\{$PR, NM$\}$ is $\mathfrak{M} = \langle W, R, \{Q(a) \mid a \in \mathrm{A}\}, V\rangle$ where

- $W = \{s \mid s$ is a maximal consistent set in PDL $\oplus$ EL $\oplus$ $\{$PR, NM$\}\}$,
- $(s, u) \in R \iff \mathcal{K}\phi \in s$ implies $\phi \in u$,
- $(s, t) \in Q(\pi) \iff [\pi]\phi \in s$ implies $\phi \in t \iff \phi \in t$ implies $\langle\pi\rangle\phi \in s$,
- $s \in V(p) \iff p \in s$.

From a similar process as in [3], it can be shown that the relation R in $\mathfrak{M}$ defined above is an equivalence relation. Please note that $\mathfrak{M}$ is a non-standard model. Please recall that a *non-standard* model for $\mathcal{L}$ is a quadruple $\langle W, R, Q, V\rangle$ that satisfies all the properties of a standard model except that, in a non-standard model, there is $Q(\pi^*) \supseteq \bigcup_{n\in\omega} \pi^n$, but it still validates all the formulas of the logic PDL $\oplus$ EL. It is easy to check that the canonical model $\mathfrak{M}$ is a non-standard model.

From a standard process shown in [3], we have the following lemma.

Lemma 4 (Truth Lemma for $\mathfrak{M}$) For each ϕ, $\mathfrak{M}, s \vDash \phi \iff \phi \in s$.

Due to the axioms PR and NM, we can show that the canonical model $\mathfrak{M}$ has the properties $com^{pr,nm}$.

Proposition 19 The canonical model $\mathfrak{M}$ has the properties of $com^{pr,nm}$.

Proof Firstly, we show that $\mathfrak{M}$ is com^{pr}. Given $(s,t) \in Q(a)$ and $(t,v) \in R$, we need to show that there is some maximal consistent set u such that $(u,v) \in Q(a)$ and $(s,u) \in R$. Let $\Phi = \{\phi \mid \mathcal{K}\phi \in s\} \cup \{\langle a\rangle\psi \mid \psi \in v\}$. We then only need to show that there exists an maximal consistent set u such that $\Phi \subseteq u$. By Lindenbaum's lemma, we only need to show that Φ is consistent. Assume that Φ is not consistent. This follows that $\vdash \mathcal{K}\phi_1 \wedge \cdots \wedge \mathcal{K}\phi_n \to [a]\neg\psi_1 \vee \cdots \vee [a]\neg\psi_k$ for some $n,k \in \omega$. Since $\vdash [a]\neg\psi_1 \vee \cdots \vee [a]\neg\psi_k \to [a](\neg\psi_1 \vee \cdots \vee \neg\psi_k)$, this follows that $\vdash \mathcal{K}\phi_1 \wedge \cdots \wedge \mathcal{K}\phi_n \to [a](\neg\psi_1 \vee \cdots \vee \neg\psi_k)$. By the generalization rule of $\mathcal{K}$ and the axioms of EL, it follows that $\vdash \mathcal{K}\phi_1 \wedge \cdots \wedge \mathcal{K}\phi_n \to \mathcal{K}[a](\neg\psi_1 \vee \cdots \vee \neg\psi_k)$. It follows by PR that $\vdash \mathcal{K}\phi_1 \wedge \cdots \wedge \mathcal{K}\phi_n \to [a]\mathcal{K}(\neg\psi_1 \vee \cdots \vee \neg\psi_k)$. Since $\mathcal{K}\phi_i \in s$ for all $1 \leq i \leq n$ and s is an MCS, this follows that $[a]\mathcal{K}(\neg\psi_1 \vee \cdots \vee \neg\psi_k) \in s$. Since $(s,t) \in Q(a)$, this follows that $\mathcal{K}(\neg\psi_1 \vee \cdots \vee \neg\psi_k) \in t$. Since $(t,v) \in R$, this follows that $\neg\psi_1 \vee \cdots \vee \neg\psi_k \in v$. This is contradictory with $\psi_i \in v$ for all $1 \leq i \leq k$. Therefore, Φ is consistent.

Secondly, we show that $\mathfrak{M}$ is com^{nm}. Given $(s,t) \in Q(a)$, $(u,v) \in Q(a)$, and $(s,u) \in R$, we need to show that $(t,v) \in R$. Take an arbitrary $\mathcal{K}\phi \in t$, then we only need to show that $\phi \in v$. Since $(s,t) \in Q(a)$, this follows that $\langle a\rangle\mathcal{K}\phi \in s$. It follows from NM that $\vdash \langle a\rangle\mathcal{K}\phi \to \mathcal{K}[a]\phi$. Thus, we have that $\mathcal{K}[a]\phi \in s$. Since $(s,u) \in R$, this follows that $[a]\phi \in u$. Since $(u,v) \in Q(a)$, this follows that $\phi \in v$. $\square$

Now we are ready to show the completeness.

Theorem 20 (Completeness)**.** $\mathrm{PDL} \oplus \mathrm{EL} \oplus \{\mathrm{PR}, \mathrm{NM}\}$ *is weakly complete w.r.t. models with* $com^{pr,nm}$.

Proof Let ϕ_0 be a consistent formula. To show the theorem, we only need to show that ϕ_0 is satisfied in a model with $com^{pr,nm}$. Next, we will show that there exists a such model.

Stage 1 Since ϕ_0 is consistent, it follows by Lindenbaum's Lemma that there is a maximal consistent set s_0 s.t. $\phi_0 \in s_0$. It follows by Lemma 4 that $\mathfrak{M}, s_0 \vDash \phi_0$.

Stage 2 Let Σ be the minimal *Fischer-Ladner* closure containing ϕ_0. We then can construct the filtration model $\mathfrak{M}^\Sigma$ as Definition 6. By Lemma 1, we have that $\mathfrak{M}^\Sigma, |s_0| \vDash \phi_0$.

Stage 3 Since $\mathfrak{M}$ has the properties of $com^{pr,nm}$(see Proposition 19), this follows that $\mathfrak{M}^\Sigma$ satisfies Propositions 9 and 10. In this stage, we will construct a coherent, saturated network of $\mathfrak{M}^\Sigma$.

Let $\Phi = \{|s| \mid (|s|, |s_0|) \in R^\Sigma\}$. It follows that Φ is finite. Let $\Phi = \{|s|_0, \cdots, |s|_n\}$ where $|s|_0 = |s_0|$. Define the network $\mathcal{N}_0$ of $\mathfrak{M}^\Sigma$ as $\mathcal{N}_0 = \langle N_0, \sim_0, \{\xrightarrow{a}_0 \mid a \in \mathrm{A}\}, L_0\rangle$ where $N_0 = \{x_0, \cdots, x_n\}$, $\sim_0 = \{(x_i, x_j) \mid x_i, x_j \in N_0\}$, $\xrightarrow{a}_0 = \emptyset$, and $L_0 = \{x_i \mapsto |s|_i \mid 0 \leq i \leq n\}$. It is obvious that $\mathcal{N}_0$ is a finite, coherent network of $\mathfrak{M}^\Sigma$.

Let $n \geq 0$ and suppose $\mathcal{N}_n$ is a finite, coherent network of $\mathfrak{M}^{\Sigma}$. Let D be the defect of $\mathcal{N}_n$ that is minimal in our enumeration. If there is no defect of $\mathcal{N}_n$ then let $\mathcal{N}_{n+1} = \mathcal{N}_n$. Otherwise, form $\mathcal{N}_{n+1}$ by repairing the defect D as described in the proof of the Repair Lemma. Observe that D will not be a defect of any network extending $\mathcal{N}_n$. Let $\mathcal{N} = \langle N, \sim, \{\xrightarrow{a} \mid a \in \mathrm{A}\}, L\rangle$ be given by

$$N = \bigcup_{n\in\omega} N_n, \sim = \bigcup_{n\in\omega} \sim_n, \xrightarrow{a} = \bigcup_{n\in\omega} \xrightarrow{a}_n, \text{ and } L = \bigcup_{n\in\omega} L_n.$$

Now we will show that $\mathcal{N}$ is coherent, and saturated. Firstly, we show that $\mathcal{N}$ is coherent. Please note that $\mathcal{N}_{n+1} \rhd \mathcal{N}_n$ and that $\mathcal{N}_n$ is finite, and coherent, for all $n \in \omega$. Thus, if $\mathcal{N}$ is not coherent, this follows that there is some $\mathcal{N}_n$ such that $\mathcal{N}_n$ is not coherent. Contradiction. Thus $\mathcal{N}$ is coherent. Secondly, we show that $\mathcal{N}$ is saturated. If it is not, let D be the minimal (according to our enumeration) defect of $\mathcal{N}$, say $D = D_k$. By our construction, there must be an approximation $\mathcal{N}_i$ of $\mathcal{N}$ of which D is also a defect. Note that D need not be the minimal defect of $\mathcal{N}_i$. There can be at most k defects that are more urgent, so D will be repaired before step $k + i$ of the construction.

Stage 4 Based on the network $\mathcal{N}$ constructed above, we can construct the model $\mathfrak{I}_{\mathcal{N}}$ as Definition 16. Since $\mathcal{N}$ is coherent, and saturated, it follows from Lemma 3 that $\mathfrak{I}_{\mathcal{N}}, x_0 \vDash \phi_0$ and from Proposition 17 that $\mathfrak{I}_{\mathcal{N}}$ has the properties of $com^{pr,nm}$. $\Box$

6 Conclusion

In this paper, we proved the weak completeness for the logic PDL$\oplus$EL extended with perfect recall and no miracles. Different from the completeness for PDL $\oplus$ EL extended with PR, no learning, or Church-Rosser Axiom, the filtration cannot automatically preserve no miracles. To tackle this problem, this paper used the step-by-step method to construct a model with no miracles based on the filtration.

References

[1] J. van Benthem, 2014, *Logic in Games*, MIT press.

[2] J. van Benthem, J. Gerbrandy, T. Hoshi and E. Pacuit, 2009, "Merging frameworks for interaction", *Journal of Philosophical Logic*, **38(5)**: 491–526.

[3] P. Blackburn, M. de Rijke and Y. Venema, 2001, *Modal Logic*, New York: Cambridge University Press.

[4] M. J. Fischer and R. E. Ladner, 1979, "Propositional dynamic logic of regular programs", *Journal of Computer and System Sciences*, **18(2)**: 194–211.

[5] D. M. Gabbay, A. Kurucz, F. Wolter and M. Zakharyaschev, 2003, *Many-dimensional Modal Logics: Theory and Applications*, North Holland: Elsevier.

[6] D. Harel, J. Tiuryn and D. Kozen, 2000, *Dynamic Logic*, Cambridge, MA, USA: MIT Press.

[7] J. Hintikka, 1962, *Knowledge and Belief: An Introduction to the Logic of the Two Notions*, Cornell University Press.

[8] M. Kracht and F. Wolter, 1991, "Properties of independently axiomatizable bimodal logics", *Journal of Symbolic Logic*, **56(04)**: 1469–1485.

[9] M. Kracht and F. Wolter, 1997, "Simulation and transfer results in modal logic: A survey", *Studia Logica*, **59(2)**: 149–177.

[10] Y. Li, 2018, "Tableaux for a combination of propositional dynamic logic and epistemic logic with interactions*", *Journal of Logic and Computation*, **28(2)**: 451–473.

[11] Y. Li, Q. Yu and Y. Wang, 2017, "More for free: A dynamic epistemic framework for conformant planning over transition systems*", *Journal of Logic and Computation*, **27(8)**: 2383–2410.

[12] R. A. Schmidt and D. Tishkovsky, 2002, "Combining dynamic logic with doxastic modal logics", in P. Balbiani, N. Suzuki, F. Wolter and M. Zakharyaschev (eds.), *Advances in Modal Logic*, **vol. 4**, pp. 371–392, King's College Publications.

[13] R. A. Schmidt and D. Tishkovsky, 2008, "On combinations of propositional dynamic logic and doxastic modal logics", *Journal of Logic, Language and Information*, **17(1)**: 109–129.

[14] R. A. Schmidt, D. Tishkovsky and U. Hustadt, 2004, "Interactions between Knowledge, Action and Commitment within Agent Dynamic Logic", *Studia Logica*, **78(3)**: 381–415.

[15] Y. Wang and Q. Cao, 2013, "On axiomatizations of public announcement logic", *Synthese*, **190**: 103–134.

[16] Q. Yu, Y. Li and Y. Wang, 2016, "A dynamic epistemic framework for conformant planning", in R. Ramanujam (ed.), *Proceedings of the 15th Conference on Theoretical Aspects of Rationality and Knowledge, TARK 2015, Pittsburgh, USA, June 4-6, 2015*, pp. 298–318, EPTCS.

[17] M. Zakharyaschev, F. Wolter and A. Chagrov, 2001, "Advanced modal logic", in D. M. Gabbay and F. Guenthner (eds.), *Handbook of Philosophical Logic*, **vol. 3**, pp. 83–266, Dordrecht: Springer Netherlands.

带有完美记忆公理和无奇迹公理的 EPDL 系统的完全性

李延军

摘　要

EPDL 系统是 PDL 和 EL 的混合系统。EPDL 的框架同时包含用于表示知识和用于表示动作的两种二元关系。完美记忆公理和无奇迹公理刻画了这两种关系的交互。本论文证明了含有完美记忆公理和无奇迹公理的 EPDL 系统相对于具有这两种交互性质的 EPDL 框架类的弱完全性。

李延军　南开大学哲学院
lyjlogic@nankai.edu.cn