Studies in Logic, Vol. 13, No. 3 (2020): 19–31
PII: 1674-3202(2020)-03-0019-13

Truth-makers for Universal Statements*

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Abstract. Fine (2016) provides some kinds of truth-maker semantics that explain how we understand our language, but these are controversial, especially in how we understand universal sentences. In his unpublished paper, Hale modifies Fine's standard exact truth-maker semantics to explain how we understand law-like universal statements. In this paper, I suggest that if Hale insists that exact truth-maker semantics clauses for law-like universal statements differ from those for accidental universal statements, it would be better for Hale to formulate a law-like generalization and an accidental generalization in different logical form, although the first entails the latter one. I also provide exact-truth-maker semantics for law-like generalization different from Hale's. In the last part of this paper, locating universal statements in inexact truth-maker semantics, I compare the two formulations of the lawlike universal statements.

1 Truth-condition Semantics and Its Different Approaches

Frege first to formulated the meaning of a sentence according to its truth conditions. In Frege's opinion, natural languages are so misleading that it quite difficult for us to explain how we understand them. For his philosophical aim, he set up a brand new logical language, transparent in its logical structure. One of his concerns was to explain the meaning of an artificial language that has atomic sentences which form all the complicated sentences are formed by the atomic sentences with connectives, quantifiers, and variables. Nowadays, it is well known that truth-condition semantics explains how truth conditions of complicated sentences depend on truth conditions of constituent sentences.

Truth-making semantics is a kind of truth condition semantics.

The main idea of truth-making is the idea of something on the side of the world — a fact, perhaps, or a state of affairs — verifying, or making true, something on the side of language or thought, a statement, perhaps or a proposition. As Fine ([5]) points out, this idea has figured prominently in contemporary metaphysics and semantics. Some philosophers, such as Armstrong, apply truth-making to arrive at a satisfactory metaphysics, asserting what on the side of the world makes statements or thoughts true. This application is a route from the language or thought side to the world side. In application of semantics, the main concern is how sentences are made true by what

Received 2019-12-05

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*This paper was supported by Renmin University of China's 2019 Fund for Building World-Class Universities (Disciplines).

is in the world. This application is a route from the side of the world to the side of language. Fine's truth-making semantics, from the very name, we know is the application of the truth-making idea to semantics.

Fine classified different approaches to truth-condition semantics. ([5]) Basically, truth-condition semantics has two kinds of approaches: the clausal approaches, the first such as suggested by Davidson ([4]), on which truth-conditions are not given as entities but by clauses specifying when a statement is true; and second, objectual approaches, according to which truth-conditions are not clauses but worldly entities that stand in truth making relation to statements they make true or false. In objectual approaches, we should also distinguish two kinds of approaches: one takes possible worlds as truth-makers, and the other takes states or situations as truth-makers. The main difference between a possible world and a state (or a situation) lies in that the former can decide any statement's truth value, while the latter cannot. For example, the state of weather in Beijing cannot settle whether it is raining in Padua. Truth-maker semantics is ojectual, taking states as truth-makers. "Possible worlds semantics" received its first systematic application to natural language in the work of Montague. ([7]) "Situations or states semantics" received its first systematic development in the work of Barwise and Perry. ([3])

Even states semantics contains further division, that is, exact truth-maker semantics, inexact truth-maker semantics, and loose truth-maker semantics. In this paper, I will focus on the first two, especially on truth-maker semantics for universal statements. But before our exploration, I provide a brief introduction to standard exact truth-maker semantics according to Fine.

2 Brief Introduction to Standard Exact Truth-maker Semantics

A state-space is a pair $\langle S, \sqsubseteq \rangle$, where S is a non-empty set of states, and $\sqsubseteq$ is a partial order on S (i.e. $\sqsubseteq$ is a reflexive, anti-symmetric and transitive relation on S). A model $\mathfrak{M}$ for the first-order language L is a quadruple $\langle S, A, \sqsubseteq, |:| \rangle$, where $\langle S, \sqsubseteq \rangle$ is a state space, A is a non-empty set of individuals, and $|:|\rangle$ is a valuation, taking each n-ary predicate G and any n individuals $a_1, \cdots, a_n$ of A into a pair (V, F) of subsets of S. Intuitively, V is the set of states that verifies G of $a_1, \cdots, a_n$ and F is the set of states that falsify G of $a_1, \cdots, a_n$. The former is also denoted by $|F, a_1, \cdots, a_n|^+$, and the latter by $|F, a_1, \cdots, a_n|^-$. It should be noted that (F, V) is not a partition of S, because $F \cup V$ might not be equal to S.

If we introduce constants a_1,a_2, $\cdots$ into the language, for each of the distinct individuals $a_1, a_n, \cdots$ in A, the clauses for exact truth-maker semantics for atomic and complex statements may be defined as follows:

(atomic)$^+$ $s \Vdash Ga_1 \cdots a_n$ if $s \in |G, a_1, \cdots, a_n|^+$
(atomic)$^-$ $s \dashv Ga_1 \cdots a_n$ if $s \in |G, a_1, \cdots, a_n|^-$

$(\neg)^+$ $s \Vdash \neg B$ if $s \dashv B$
$(\neg)^-$ $s \dashv \neg B$ if $s \Vdash B$
$(\wedge)^+$ $s \Vdash B \wedge C$ if for some states, $t, u, t \Vdash B, u \Vdash C$, and $s = t \sqcup u$
$(\wedge)^-$ $s \dashv B \wedge C$ if $s \dashv B$ or $s \dashv C$
$(\vee)^+$ $s \Vdash B \vee C$ if $s \Vdash B$ or $s \Vdash C$
$(\vee)^-$ $s \dashv B \vee C$ if for some states $t, u, t \dashv B$, $u \dashv C$, and $s = t \sqcup u$
$(\forall)^+$ $s \Vdash \forall x \phi(x)$, if there are states $s_1, s_2, \cdots$ with $s_1 \Vdash \phi(a_1), s_1 \Vdash \phi(a_2), \cdots$, and $s = s_1 \sqcup s_2 \sqcup \cdots$
$(\forall)^-$ $s \dashv \forall x \phi(x)$, for some $a \in A$, $s \dashv \phi(a)$
$(\exists)^+$ $s \Vdash \exists x \phi(x)$, for some $a \in A$, $s \Vdash \phi(a)$
$(\exists)^-$ $s \dashv \exists x \phi(x)$, if there are states $s_1, s_2, \cdots$ with $s_1 \dashv \phi(a_1), s_1 \dashv \phi(a_2), \cdots$, and $s = s_1 \sqcup s_2 \sqcup \cdots$

3 Some Features of Exact Truth-maker Semantics

3.1 States in semantics

The states in Fine's standard truth-maker semantics are just terms of art that need not stand for a state in any intuitive sense of the term.

> It should be noted that our approach to states is highly general and abstract. We have formed no particular conception of what they are; and nor have we assumed that there are atomic states, from which all other states can be obtained by fusion. ([5], p. 4)

Fine ([5]) suggests distinguishing truth-makers in metaphysics and semantics. He does not agree that the truth-making idea provides a guide to metaphysics. Fine's truth-maker semantics, as he says, concerns how truth-makers (states of affairs) make statements in our language true.

> The idea of truth-making is the idea of something on the side of the world, a fact, perhaps, or a state of affairs, verifying, or making true, something on the side of language or thought, a statement, perhaps, or a proposition. The idea of truth-making has figured prominently in contemporary metaphysics and semantics. In its application to metaphysics, the thought has been that we can arrive at a satisfactory metaphysical view by attempting to ascertain what it is, on the side of the world, that renders true what we take to be true (as in [1] and [2]); and, on the semantical side, the thought has been that we can attain a satisfactory semantics for a given language by attempting to ascertain how it is that the sentences of the language are made true by what is in the world. In the former case, truthmaking serves as a conduit taking us from language or thought to an

> understanding of the world; In the latter case, it has served as a conduit taking us from the world to an understanding of language. ([5], p. 4)

3.2 Exact truth-makers

According to Fine, the exact truth-maker of a statement should be wholly relevant to the statement.

Both exact and inexact verification require a relevant connection between state and statement. With inexact verification, the state should be at least partially relevant to the statement, and with exact verification, it should be wholly relevant. Thus, the presence of rain will exactly verify the statement 'it is rainy'; the presence of wind and rain will inexactly verify for the statement 'it is rainy', though not an exact verifier; and the presence of wind will be a loose verifier for the statement 'it is rainy or not rainy' (since the statement is true no matter whether rain is present), while failing to be an inexact verifier.

However it is wrong to think that an exact truth-maker of a statement is the minimal state that makes it true. Let us consider $p \vee (p \wedge q)$. According to exact truth-maker semantics,

- $s \Vdash p \vee (p \wedge q)$ if $s \Vdash p$ or $s \Vdash p \wedge q$;
- $s \Vdash p \wedge q$ if for some states t, u, $t \Vdash p, t \dashv q$, and $s = t \sqcup u$;
- therefore, the exact verifier of p and the exact verifier of $p \wedge q$ are both exact truth-makers of $p \vee (p \wedge q)$, although the latter includes the first.

From this example, it is easy to see that a state could be an exact truth-maker for different statements[1], and a statement may have different exact truth making states.[2]

4 Quantifiers

4.1 Two kinds of generalization

As for universal statements, Hale urges us to notice the difference between two kinds of generalization. To illustrate the difference, let us consider the two statements in the following:

Example 1

- All cats are born blind.
- All the students in my logic class are male.

The first statement is law-like, and we might think that there are no contractual instances for this universal statement; the second statement is accidental, and we

[1] In this example, s is not only the exact verifying statement of p, but also of $p \vee (p \wedge q)$.

[2] In this example, $p \vee (p \wedge q)$ may have different states exactly verifying its truth.

might think of contractual instances of the universal statement. We represent these thoughts as the following statements:

Example 2

- If the cat were justly born, he would have been blind.
- If Alice were in my logic class, she would have been male.

These two examples may show the difference between the two kinds of generalization. However the standard exact truth-maker semantics for them are same. Hale thinks that an adequate semantics for generalization should tell the difference between the two kinds. But Fine's standard truth-maker-semantics cannot account for the difference, and therefore we should fix that.

4.2 Hale's Way

Hale does not suggest that the two universal statements in example 1 are in different logical forms, but he insists that although they are in the same logical form, the clauses for exact truth-makers in semantics for the two kinds of general statements should differ. The law-like and accidental universal statements have the same logical form: $\forall x(Hx \rightarrow Gx)$. For the exact truth-maker semantics of accidental universal statements, Hale, suggests we should extend the valuation $|:|$ to assign to the pair $\langle H, G\rangle$ a pair $\langle \mathfrak{V}^{H,G}, \mathfrak{F}^{H,G}\rangle$, the former comprising states which verify G of any n-tuple $a_1, a_2, \cdots a_n$ also verify F and the second comprising those states which falsify G of any n-tuple $a_1, a_2, \cdots a_n$ but verify F. We may denote the set of states of the first kind by $|\langle H, G\rangle|^+$, and the second by $|\langle H, G\rangle|^-$.

Hale's modification of exact truth-maker semantics is a kind of restriction of the model $\langle S, A, \sqsubseteq, |:|\rangle$. Firstly, we collect all individuals for s verifying F of a. Secondly, for s if it is also the verification of G for all the collected individuals, then $s \in |\langle H, G\rangle|^+$. For s, if s is a falsification of G for some collected individual, then $s \in |\langle H, G\rangle|^-$. Exact truth-maker semantics for accidental universal statements essentially differ from law-like ones. From the above, we see that this semantics for accidental ones does not resort to the meaning of material implication. To contrast with the law-like ones, it is better to formulate the accidental ones in the form $\forall x(Hx \prec Gx)$

For the law-like universal statements, Hale asks for different requirements. To simplify the explanation, it is better to introduce the connective $\rightarrow$. Like classical semantics for material implication, Hale introduces $H \rightarrow G$ as the abbreviation for $\neg H \vee G$.

$(\rightarrow)^+$ $s \Vdash H \rightarrow G$ if $s \Vdash \neg H$ or $s \Vdash G$
$(\rightarrow)^-$ $s \dashv H \rightarrow G$ if for some t, u, $t \Vdash$ and $u \dashv C$ and $s = t \sqcup u$.

To make the explanation more easier, let F and G be monadic predicate.

Hale suggests that if s is a verifier of the law-like universal statement $\forall x(Hx \rightarrow Gx)$, then s should be the verifier for the open statement $Hx \rightarrow Gx$, no matter which individual a in the domain is taken as the value of x. If a state verifies or falsifies the open statement, then the state is a generic state.

It is easy to prove that if s is a general state verifying or falsifying a law-like universal statement, it is also a verifying or falsifying the universal statement as accidental statement.

5 My Modifications for Quantifiers

I agree with Hale that semantics should be adequate to account for the semantic difference between law-like and accidental universal-statements. I am also in sympathy with Hale's proposal against taking the law-like generalization in necessary form. Drewery proposes that the law-like universal statement should have a logical form different from the accidental one's, and her suggestion for the law-like involves necessity, and might be represented as the logical form $\Box\forall x((Hx \wedge Gx) \leftrightarrow Gx)$. Here I do not wish to explain in detail why this kind of suggestion involving necessity like Drewery's is not attractive. But the main reason I do not take this suggestion is that here, we wish to provide the exact verifier or falsifier of law-like universal statements, and the truth-maker for necessity is loose but not exact. Unless there are no exact truth-makers for universal statements, we do need to consider the exact truth-makers for the universal statements. There is no sufficient reason to deny the exact truth-makers of universal statements.

No matter what logical form represents a law-like universal statement, it entails $\forall x(Hx \rightarrow Gx)$. The main concern is how to explain semantic conditions for universal statements and more special requirements for law-like ones. I do not mean that the two statements in example 1 are in the same logical form, instead, I suggest, since their semantic requirements differ, they are take different logical forms. Hale suggests the two are in the same logical form, but at the same time gives different semantic clauses for the two kinds of universal statements. It might not be a big deal since we wish to explain the two kinds of universal statements, and even if we use the same symbols representing the two kinds of statements, the semantic clauses differ. Of course, once we explain the form in two different ways, the one form essentially represents two different forms.

Naturally, as for an accidental universal-statement, like "All the students in my logic class are male," if we say that s is the exact verifier of this statement, we expect that s includes state t verifying "a_1 is in my logic class; $\cdots$ a_n is in my logic class", includes some other states respectively verifying "a_1 is in my logic class, $\cdots$, a_n is in my logic class". The fusion of all the states is s. The fusion of the states is an upper-bound of these states, respective to the partial order $\sqsubseteq$ in semantics. It should also be noticed that the exact verifier is not hereditary in the sense that if s is an exact

verifier for a statement, and t includes s, then generally t is not an exact verifier of the statement.

In Hale's modification, an exact verifier s for a universal statement should be the exact verifier of all the instances. But there is no good reason why s should be the exact verifier for the instances but not inexact instead. For an accidental statement, like "Tom is in my logic class," "John is in my logic class." Exact verifiers for the two statements are not supposed to have an internal relationship, and the fusion of the two verifiers is the conjunction of the two statements. However in Hale's modification, if a state is an exact of some universal statement, no matter whether it is law-like or accidental, this state at least includes exact verifiers for the instances. Fatherly, the verifier for an accidental universal-statement should also include a state t collecting all individuals that F holds of. To decide whether it is true that all students in my logic class are male, I should see which are in my class and whether they are male. Therefore, t collecting all individuals that F holds, should be part of the verifier. Hale suggests extending the valuation is just to consider the state collecting all the individuals that F holds. Therefore, I almost agree with his modification; the minor difference is that I require only that the verifier include verifiers for instances. Briefly, I suggest adopting Fine's exact truth-maker semantics for $\forall$, and explain accidental "All As are Bs" just by restriction to domain. I formulate this logical form as $\forall x(A(x) \Subset B(x))$.

I also agree with Hale that the law-like universal statements might not concern the domain of a model. "All cats are born blind," no matter whether the cats are deceased, alive, or born in the future. As for this statement, we might not even collect all individuals being cat. Hale proposes that if s is a verifier for the law-like universal in the form $\forall x(Hx \rightarrow Gx)$ should be a generic statement that is a verifier for the open statement in the form $Hx \rightarrow Gx$. My worry is the semantics for the connective of material implication.

The exact verifier "All cats are born blind" should also verify "Tom is not a cat, and he is not born blind" or "Cooky is a monkey, and he is unluckily born blind", and etc. In Hale's modification for the law-like, verifiers also take all individuals in the domain into account. Furthermore, the generic state is not only the verifier for this universal statement but also an exact verifier for "any individuals being cat are then being born blind."

In my opinion, the law-like universal statement "all Fs are Gs" is about the relation between two concepts rather than individuals. I think Hale also agrees with that. He says:

> For as I there suggested, even in cases where the domain of quantification is finite, but open ended, as with many ordinary non-accidental true generalizations, there is reason to doubt the availability of an instance-based explanation of their truth. If one accepts that such generalizations

> are true (and not merely that they will eventually — when the world ends — be true), and one further accepts that the states which make their instances true exist only if, and when, the objects involved in them exist, and that instances concerning as yet non-existent objects, even if stateable, cannot be made true by states which do not (yet) exist, then there is reason to doubt that those generalizations can be made true by states which are the fusions of states verifying each of their individual instances. ([6])

The point is that, Hale's resolution does not realize his wish. As I said before, in Hale's modified semantics, the verifier of a law-like universal statement should take into account all individuals in the domain, and they cannot be beyond the domain. But I do not think it is promising to realize the wish in model theoretic semantics, as in the true-maker semantics. The main reason is that the universal quantifier is explained by the notion of domain. Unless we reject this idea, there is no hope to realize our wish.

Hale interprets the connective in $H \rightarrow G$ as the abbreviation for $\neg H \vee G$. I would rather interpret it like this: The exact verifier of F includes the exact verifier of G. Inspired by [5] (in part II Application, §1), I first formulate semantic clauses for $H \leftrightarrow G$, and then introduce $H \rightarrow G$ by abbreviation for $H \wedge G \leftrightarrow H$.

$(\leftrightarrow)^+$ $s \Vdash H \leftrightarrow G$ if $s \Vdash H$ iff $s \Vdash G$
$(\leftrightarrow)^-$ $s \dashv H \leftrightarrow G$ if $s \Vdash H$ but$\dashv G$ or $s \Vdash G$ but $\dashv H$

We define a verifier or falsifier of a statement by induction on its structure, and intuitively, truth-makers have some structure that also decides whether a state is a truth-maker of a given statement. For example, the verifier s of $A \wedge B$ should also witness the structure of states, for it requires that s include an exact verifier of B. Generally speaking, truth-maker semantics of complicated statements concerns the structure of states.

Intuitively, s is a verifier of $A \leftrightarrow B$ if (s is a verifier of A if and only if s is a verifier of B). Notice: These clauses for verifiers or falsifiers are defined by induction, and they are nested. As for $\neg A$, its verifier is just a falsifier of A. Therefore the induction is two lateral, in contrast with the usual semantics, which is unilateral. Generally, $\nVdash G$ does not mean $\Vdash \neg G$ or $\dashv G$.

As for law-like universal statements in the form of $A \rightarrow B$, clauses could be formulated as in the following:

$(\rightarrow)^+$ $s \Vdash A \rightarrow B$ if $s \Vdash (A \wedge B) \leftrightarrow A$
$(\rightarrow)^-$ $s \dashv A \rightarrow B$ if $s \dashv (A \wedge B) \leftrightarrow A$

We could infer semantic clauses for law-like universal statements in the form $\forall x(Hx \rightarrow Gx)$ as in the following:

$(\forall x)^+$ $s \Vdash \forall x(Hx \to Gx)$ if for all $a \in A$, $s \Vdash H(a) \to G(a)$
$(\forall x)^-$ $s \dashv \forall x(Hx \to Gx)$ if for some $a \in A$, $s \dashv H(a) \to G(a)$

Although prima facie, here, semantic clauses for law-like universal statements are the same as Hale's, they differ because the semantic clauses for $\to$ are different.

6 Comparison Between Hale's and My Semantics for Law-like Universal Statements

6.1 The difference

To explain the difference between Hale's and my semantic for law-like universal statements, let us consider the verifier s of $\forall x(Fx \to Gx)$ in Hale's semantics:

$s \Vdash F(x) \to G(x)$ if for all $a \in A$, $s \Vdash \neg F(a)$ or $s \Vdash G(a)$.

Now let $s \Vdash F(a)$, if $\forall x(Fx \to Gx)$; then, because of the exclusiveness of semantics(s cannot be both verifier and falsifier of any statement), $s \vdash G(a)$.

However, in my semantics, this is not necessarily so. Again let $s \Vdash F(a)$, if $\forall x(Fx \to Gx)$; then, $s \Vdash F(a) \wedge G(a)$. Therefore, s should include an exact verifier t, $t \Vdash G(a)$. From this, we see s should not necessarily be the exact verifier of B.

This example sufficiently shows that Hale's and my semantics differ.

6.2 The strength of the four logical forms

To make the explanation easier, in my modification, the law-like universal statement's logical form is $\forall x(Hx \hookrightarrow Gx)$.

It is natural to ask the strength order of the four logical forms: $s \Vdash \forall x(Hx \to Gx)$, $s \Vdash \forall x(Hx \hookrightarrow Gx)$, $s \Vdash \forall x(Hx \prec Gx)$, $s \Vdash Hx \Subset Gx)$.

Claim 1 $s \Vdash \forall x(Hx \hookrightarrow Gx)$ and $s \Vdash \forall x(Hx \to Gx)$ cannot infer each other.

Proof Let $s \Vdash \forall x(Hx \to Gx)$. From the requirement in Hale's semantics, $s \dashv H(a)$ or $s \Vdash G(a)$ for all $a \in A$. If $s \dashv H(a)$, then $s \Vdash G(a)$. So, for all $a \in A$, if $s \Vdash H(a)$, then $s \Vdash H(a) \wedge G(a)$. This does not mean $s \Vdash \forall x(Hx \to Gx)$ can infer $s \Vdash \forall x(Hx \hookrightarrow Gx)$,we need another direction. For any $a \in A$, if $s \Vdash H(a) \wedge G(a)$, and $s \Vdash \forall x(Hx \to Gx)$, then for some t, u, $t \Vdash H(a)$, $u \vdash G(a)$, and $s = u \sqcup t$. But we cannot infer $s \Vdash H(a)$. Therefore, we cannot infer $s \Vdash \forall x(Hx \hookrightarrow Gx)$.

Now let $s \Vdash \forall x(Hx \hookrightarrow Gx)$. For any $a \in A$, if $s \Vdash H(a)$, then $s \Vdash H(a) \wedge G(a)$. Neither could we infer $s \Vdash G(a)$. Therefore we cannot infer $s \Vdash \forall x(Hx \to Gx)$ from $s \Vdash \forall x(Hx \hookrightarrow Gx)$.

From the above, we cannot compare the strength between $s \Vdash \forall x(Fx \hookrightarrow Gx)$ and $s \Vdash \forall x(Fx \to Gx)$. □

Claim 2 $s \Vdash \forall x(Fx \hookrightarrow Gx)$ entails $s \Vdash \forall x(Hx \prec Gx)$.

Proof Now let $s \Vdash \forall x(Fx \hookrightarrow Gx)$. For any $a \in A$, if $s \Vdash F(a)$, then $s \Vdash F(a) \wedge G(a)$. s is **also** the exact verifier of instances of $F(a)$; then s should exactly verify these instances being G. Therefore, $s \Vdash \forall x(Fx \hookrightarrow Gx)$ entails the corresponding accidental universal-statement $s \Vdash \forall x(Hx \prec Gx)$. □

Claim 3

(i) $s \Vdash \forall x(Hx \rightarrow Gx)$ entails $s \Vdash \forall x(Hx \prec Gx)$.
(ii) $s \Vdash \forall x(Fx \prec Hx)$ entails $s \Vdash \forall x(Fx \Subset Hx)$.

Proofs for the two claims are very easy.

7 Inexact Truth-makers

7.1 Hereditariness

Exact truth-makers are not hereditary in the sense that if $t \Vdash A$, and $t \sqsubseteq s$, then s may not be an exact verifier of A. Exact truth-maker $s \Vdash A \wedge B$ does not entail $s \Vdash A$, or $s \Vdash B$. The following example may illustrate the feature well:

$t_1 \Vdash A$ and $t_2 \Vdash \neg A$, $t_1 \sqcup t_2 = s$. But s is not exact verifier or falsifier of A. Although $s \Vdash A \wedge \neg A$, this does not entail inconsistency in the semantic theory of truth-makers. s is a fusion of t_1 and t_2, although t_1 and t_2 are incompatible. The basic rule for semantics is that a statement cannot be both an exact verifier and a falsifier of a statement. This example does not break the rule, for s is just an exact verifier of $A \wedge \neg A$, but not an exact verifier both of A and of $\neg A$.

Unlike exact truth-makers, the inexact truth-maker is hereditary. Before I explain the hereditariness of inexact truth-maker semantics, Fine's inexact truth-maker semantics should first be introduced.

Generally, an exact truth-maker s of statement A is also an inexact truth-maker of A. The exact truth-maker should be relevant as a whole to the statement that it makes true, but the inexact truth-maker should be relevant either in part or as a whole to the statement that it makes true.

$\wedge$: A state is an inexact truth-maker for the conjunction $A \wedge B$ iff it is an inexact truth-maker for each conjunct B and C.
$\vee$: A state is a falsifier for the disjunct $A \vee B$ iff it is a falsifier for A and B.
$\forall x$: A state is a truth-maker for the universal quantification $\forall x A(x)$ iff it is a truth-maker for each $A(a_1), A(a_2), \cdots$.
$\exists x$: A state is a falsifier for $\exists x A(x)$ iff it is a falsifier for each-maker for each $A(a_1), A(a_2), \cdots$.

Clauses for negation, negative clauses for conjunction and universal quantification and positive clauses for disjunction and existential quantification are the same as exact semantics.

Let us state some basic rules for inexact semantics:

(i) (Exclusiveness) The state cannot be both a verifier and a falsifier of a statement.
(ii) (Hereditariness) If statement t is a truth-maker of statement A, and t is a part of state s, then s is a truth-maker of A.

7.2 Some basic features of connectives in inexact truth-maker semantics

From the inductive definition of the inexact truth-maker for the statements, there are some basic results in this semantics:

(i) There is no s, such that $s \Vdash A \wedge \neg A$.
(ii) There is no s, such that $s \Vdash \neg(A \vee \neg A)$.
(iii) If s is an exact truth-maker of statement A, then s is an inexact truth-maker of A.

7.3 Reconsider the law-like universal in inexact truth-maker semantics

In this semantics, let us reconsider the law-like universal statement in inexact truth-maker semantics. Following Hale's proposal,

$s \Vdash \forall x(Fx \rightarrow Gx)$ if for all $a \in A$, $s \Vdash \neg Fa$ or $s \Vdash \neg Ga$.

Claim 4 $s \Vdash \forall x(Fx \rightarrow Gx)$ entails $s \Vdash \forall x(Fx \hookrightarrow Gx)$.

Proof Suppose $s \Vdash \forall x(Fx \rightarrow Gx)$. For any $a \in A$, if $s \Vdash Fa$, then s is not a verifier of $\neg Fa$. By Hale's definition, $s \Vdash Ga$, so $s \Vdash Fa \wedge Ga$.

Suppose $s \Vdash \forall x(Fx \rightarrow Gx)$. For any $a \in A$, if $s \Vdash Fa \wedge Ga$, then $s \Vdash A$.

Therefore $s \Vdash \forall x(Fx \hookrightarrow Gx)$. □

Suppose $s \Vdash \forall x(Fx \hookrightarrow Gx)$. For any $a \in A$, s is a verifier of Fa iff s is a verifier of $Fa \wedge Ga$. Then, for any $a \in A$, if s is not a verifier of $\neg F(a)$, then s is not necessarily a verifier of $F(a)$. At the same time, s is not necessarily a verifier of Ga.

Example 3 A is the domain of a model, and s is a state of this model. But s is not a verifier of $F(a_8)$,$G(a_8)$ nor a falsifier $F(a_8)$. But for any $a \in A$, s is a verifier of Fa iff s is a verifier of $Fa \wedge Ga$, so $s \Vdash \forall x(Fx \hookrightarrow Gx)$. At the same time s is not a verifier of $\neg F(a_8)$ and not a verifier of $G(a_8)$. Therefore s is not a verifier of $\forall x(Fx \rightarrow Gx)$.

This example shows that in inexact semantics, $s \Vdash \forall x(Hx \hookrightarrow Gx)$ does not entail $s \Vdash \forall x(Hx \rightarrow Gx)$.

Definition 4 (Exhaustiveness) If the valuation $|:|$ of a model satisfies the condition: for any statement A, for any state in the model, s is verifier or falsifier of A, then the valuation or the model is exhaustive.

Claim 5 $\mathfrak{M}$ is a model, and s is a state in the model. If $\mathfrak{M}$ is exhaustive, then in the inexact semantics $s \Vdash \forall x(Hx \hookrightarrow Gx)$ entails $s \Vdash \forall x(Hx \rightarrow Gx)$.

Proof Suppose $s \Vdash \forall x(Hx \hookrightarrow Gx)$. For any $a \in A$, s is a verifier of Ha iff s is a verifier of $Ha \wedge Ga$. Then, for any $a \in A$, if s is not a verifier of $H(a)$, then by exhaustiveness, it is a falsifier of $H(a)$, that is, $s \Vdash \neg A$, so $s \Vdash H(a) \rightarrow G(a)$. If s is a verifier of $H(a)$, then by definition of $s \Vdash \forall x(Hx \hookrightarrow Gx)$, $s \Vdash H(a) \wedge G(a)$. By the definition of $\wedge$, $s \Vdash G(a)$, so $s \Vdash H(a) \rightarrow G(a)$. Therefore $s \Vdash \forall x(Hx \rightarrow Gx)$. □

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全称陈述句的使真者

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摘　要

Fine（2016）给出了几种使真者语义学来解释我们如何理解我们的语言，但是它们是有争议的，特别在我们如何理解全称句子方面存在争议。在 Hale 未发表的论文中，Hale 修正了严格使真者的语义学，来解释我们如何理解规律性的全称陈述句。在这篇文章中，我认为如果 Hale 坚持规律性的全称句子和偶然性的全称句子在严格使真者语义上的区分，那么最好区分这两种全称句子的逻辑形式，尽管前一种全称句子蕴涵后一种全称句子。我还给出了另一种规律性全称句子的语义条件来区于 Hale 所给出的语义条件。在文章的最后一部分，我在非严格的使真者语义学中比较了这两种规律性的全称句子的不同语义条件。

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