

Indicative Conditionals as Epistemic Filter Conditionals*

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Abstract. Deng & Lee (2021) recently give an “extrapolationist” causal-modeling account of the semantics of indicative conditionals. It requires us to have a priority relation on variables and to calculate the induced ordering for submodels, in terms of which the set of minimally extrapolated submodels is defined to account for the semantics of indicative conditionals. In this paper, the authors argue that this extrapolationist proposal has unnecessary complexity and may yield incorrect results first, and then offer an “epistemic filter” account of indicative conditionals that, the authors believe, can successfully avoid these problems.

1 Introduction

There are two different kinds of conditionals in English: indicative and subjunctive.¹ An OK pair, as called by [4], is a pair of an indicative conditional and a subjunctive conditional that have the same antecedent and succedent yet differ in their truth-values. In what follows, we use “ $\varphi \rightarrow \psi$ ” and “ $\varphi > \psi$ ” to stand for, respectively, an indicative conditional and a subjunctive conditional. The standard example of an OK pair is the following one (the intuitive truth-values of which are attached behind them):²

- (1a) If Oswald did not kill Kennedy, someone else did. (T)
- (1b) If Oswald had not killed Kennedy, someone else would have. (F)

Notice that the truth of (1a) is especially obvious when an investigation was going on without reaching the conclusion that it was Oswald who killed Kennedy, while the

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¹It is assumed in this paper that they differ not only grammatically, but also, though not everyone will agree, semantically. E.g., R. Stalnaker ([11]) and D. Edgington ([4]), will prefer a semantically unified semantic treatment for both kinds of conditionals.

²This standard example is from E. Adams. ([1])

falsity of (1b) is especially obvious when the investigation was over and the conclusion reached was that it was Oswald who killed Kennedy. Since (1a) and (1b) have the same antecedent and succedent but differ in their truth-values, the operators contained in them, i.e., “ $\rightarrow$ ” and “ $>$ ”, must differ in meaning. The problem remains: how should we account for this semantic difference?

There are several answers to this question, the most popular one no doubt being the one proposed by [8] and [7] which takes an indicative conditional to be a material implication and interprets a subjunctive conditional in terms of the closest-possible-world semantics. Our focus in this paper, however, is on another answer that was recently proposed by [3]. In terms of causal models, [3] proposes the following answer to our question: a subjunctive conditional should be interpreted as an interventionist conditional as proposed by [2, 5, 6, 12], while an indicative conditional should be understood as an “extrapolationist conditional” of their proposal. To be more specific, [3] proposes that (to be explained in more details in the next section):

- (CC) “ $\varphi > \psi$ ” is true in a causal model M , if and only if, “ ψ ” is true in the submodel $M_{(\varphi)}$.
- (IC) “ $\varphi \rightarrow \psi$ ” is true in a causal model M relative to a context c iff either (i) $\mathbb{M}^{[\varphi]}$ is empty, or (ii) “ ψ ” is true in all minimal extrapolated submodels in $\mathbb{M}^{[\varphi]}$ as ordered by the priority relation provided by c .

Notice that (CC) is defined w.r.t. a model, while (IC) is defined w.r.t. both a model and a context c .

We will side with [3] in accepting (CC), but we don’t think that it is a good idea to accept (IC). We will explain the main ideas contained in (IC) in Section 2, point out several problems of accepting (IC) in Section 3, and then propose our own account of the semantics for indicative conditionals in Section 4.

2 The Semantics for (IC) Proposed by Deng & Lee

Beside (1a) and (1b), [3] gives two more examples of OK pair. Here is one of them (Imagine that a fair coin was tossed and landed head, but you had bet tail. For the intuitive falsity of (2a), imagine that it was asserted before the toss and before you bet; for the intuitive truth of (2b), imagine that it was asserted after the toss and after you bet):

- (2a) If you bet head, you will win. (F)
- (2b) If you had bet head, you would have won. (T)

Here is another example (Imagine: you know that the local weather is such that if the atmospheric pressure is low, a rainstorm will be approaching, and that if otherwise, then a sandstorm will be approaching. Moreover, you know that you will see a black pillar of cloud in either case. For the intuitive truth of (3a), imagine that it was as-

serted before you saw a rainstorm; for the intuitive falsity of (3b), imaging that it was asserted after you saw a rainstorm³):

- (3a) If a rainstorm is not approaching, I will see a black pillar of cloud. (T)
- (3b) If a rainstorm had not been approaching, I would have seen a black pillar of cloud. (F)

These three examples and their accompanied intuitive truth-values are the only OK pairs that [3] have considered and tried to capture by their account. As we mentioned in the beginning of this paper, their account consists of the following pair of stipulations of truth-conditions for subjunctive conditionals (CC) and for indicative conditionals (IC):

- (CC) “ $\varphi > \psi$ ” is true in a causal model M , if and only if “ ψ ” is true in the submodel $M_{(\varphi)}$.
- (IC) “ $\varphi \rightarrow \psi$ ” is true in a causal model M relative to a context c iff either (i) $\mathbb{M}^{[\varphi]}$ is empty, or (ii) “ ψ ” is true in all minimal extrapolated submodels in $\mathbb{M}^{[\varphi]}$ as ordered by the priority relation provided by c .

We assume that readers of this paper are already familiar with the interventionist account of subjunctive conditionals, so that if they apply (CC) to cases (1b)-(3b), they will see that all of them have the intuitive truth-valued according to (CC). So far, their account seems to be right; the problem, however, is whether (IC) captures the intuitive truth values of (1a)-(3a)?

In order to see the merit of their account, let us briefly explain what are involved in (IC). First, according to [3], a causal model $M = \langle U, V, F, A \rangle$ satisfies the following three conditions:

- (i) U and V are finite sets of variables, which are called “exogenous variables” and “endogenous variables” respectively;
- (ii) F is a set of structural equations specifying the causal relationships among variables. For each $V_i \in V$, F contains exactly one structural equation which tells us how V_i is a function of its “parents”;
- (iii) A assigns values to each member of U in the model.

Second, if $M = \langle U, V, F, A \rangle$, then $\mathbb{M}^{[\varphi]}$ is the set of all models $\langle U, V, F, A^* \rangle$ that are “extrapolated from M ”, i.e., it is the set of all models that keep U, V and F unchanged but satisfy the condition that φ is true according to A^* . Third, since there may be more than one models in $\mathbb{M}^{[\varphi]}$, it is desirable (and actually necessary) to order them by some “priority relation” provided by a context c , otherwise, we won’t get the desired truth-values when evaluating (1a)-(3a). Fourth, the way a context c provides the priority relation is via two complex procedures: first, the context c orders variables in $U \cup V$

³We actually do not have a strong intuition that (3b) should be false. However, for the purpose of this paper, we will assume that Deng & Lee are right in claiming that (3b) is false when uttered after you saw a rainstorm.

by a total preorder; second, with this priority relation among variables in hand, the context c then determines a total preorder for extrapolated submodels in $\mathbb{M}^{[\varphi]}$ by some kind of lexicographical order. The idea is that, when we compare the extrapolated submodels, we start with those variables that have a top priority to be held fixed, so that any submodel that diverges from actual M at these variables should be judged as more deviant than others. A extrapolated model in $\mathbb{M}^{[\varphi]}$ which is not more deviant than any other model in $\mathbb{M}^{[\varphi]}$ is called “a minimal extrapolated submodel” in $\mathbb{M}^{[\varphi]}$. Finally, we use the set of minimal extrapolated submodels, denoted by $\min(\mathbb{M}^{[\varphi]})$, to determine whether “ $\varphi \rightarrow \psi$ ” is true in M relative to context c .

In short, in order to see what truth-value that an indicative conditional “ $\varphi \rightarrow \psi$ ” has relative to a model M and a context c , Deng & Lee’s proposal requires us to go through the following procedure:

- Step 1. $[M + \varphi] \Rightarrow \mathbb{M}^{[\varphi]}$; i.e. use the model M and the antecedent φ to determine the extrapolated set $\mathbb{M}^{[\varphi]}$ of submodels.
- Step 2. $[\mathbb{M}^{[\varphi]} + c] \Rightarrow \text{priority relation} \Rightarrow \min(\mathbb{M}^{[\varphi]})$; i.e., use $\mathbb{M}^{[\varphi]}$ and the context c to determine, via a priority relation offered by c , the set of minimal extrapolated models relative to c .
- Step 3. $[M, c, \min(\mathbb{M}^{[\varphi]})] \Rightarrow$ evaluate the truth-value of “ $\varphi \rightarrow \psi$ ” in M relative to c by (IC).

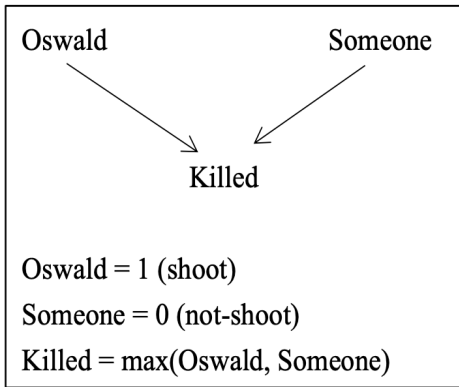


Diagram 1-1

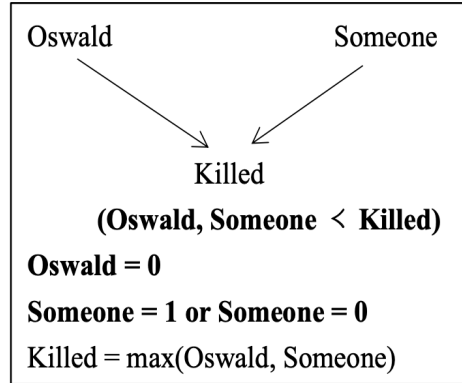


Diagram 1-2

A brief revisit of the previous three examples will allow us to see how Deng & Lee’s proposal works for them. For (1a), the model $M_1 = \langle U_1, V_1, F_1, A_1 \rangle$ in question is such that $U_1 = \{\text{Oswald, Someone}\}$, $V_1 = \{\text{Killed}\}$, $F_1 = \{\text{Killed} = \max(\{\text{Oswald, Someone}\})\}$ and A_1 is such that $A_1(\text{Oswald}) = 1$ and $A_1(\text{Someone}) = 0$, as indicated by Diagram 1-1.

In order to evaluate the indicative conditional (1a), we go through the 3 steps mentioned above. After step 1, it can easily be seen that there are two extrapolated

models in $\mathbb{M}^{[\text{Oswald}=0]}$, as indicated by “Oswald = 0” and “Someone = 1 or Someone = 0” in Diagram 1-2. If all these models in $\mathbb{M}^{[\text{Oswald}=0]}$ were in $\min(\mathbb{M}^{[\text{Oswald}=0]})$, then (1a) would not be evaluated as true, as desired, in M_1 relative to c (because the extrapolated model that assigns 0 to both Oswald and Someone will assign 0 to Killed too). Hence the context c comes in as a rescuer; it takes the fact that Kennedy was killed to be the most important fact to “be fixed” or to be presupposed by the speakers in the context (don’t ask why), as indicated in Diagram 1-2 by “Oswald, Someone $\prec$ Killed”, and orders the two models accordingly so that the minimal one is the one that assigns 1 to Someone (because the other extrapolated model will deny this important fact). Since the succedent of (1a) is also true in this unique minimal model in $\mathbb{M}^{[\text{Oswald}=0]}$, the indicative conditional (1a) is evaluated as true, as desired.

For (2a), the model $M_2 = \langle U_2, V_2, F_2, A_2 \rangle$ in question is such that $U_2 = \{\text{Bet}, \text{Land}\}$, $V_2 = \{\text{Win}\}$, $F_2 = \{\text{Win} = (\text{Bet} = \text{Land})\}$ and A_2 is such that $A_1(\text{Bet}) = 0$ (tail) and $A_2(\text{Land}) = 1$ (head), as indicated by Diagram 2-1.

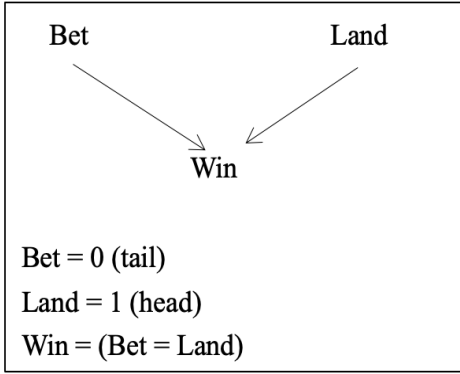


Diagram 2-1

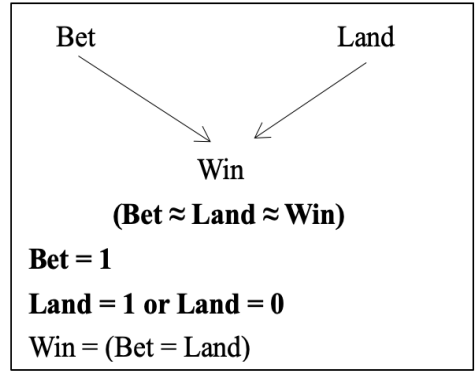


Diagram 2-2

Again, in order to evaluate the indicative conditional (2a), we go through the 3 steps mentioned above. After step 1, it is easy to see that there are two extrapolated models in $\mathbb{M}^{[\text{Bet}=1]}$, as indicated by “Bet = 1” and “Land = 1 or Land = 0” in Diagram 2-2. This time, however, the context presupposes no fact at all, as indicated by “Bet $\approx$ Land $\approx$ Win”, so that all these models in $\mathbb{M}^{[\text{Bet}=1]}$ are also in $\min(\mathbb{M}^{[\text{Bet}=1]})$. Since the succedent of (2a) is not true in one of these minimal models in $\mathbb{M}^{[\text{Bet}=1]}$ (i.e., not true in the extrapolated model that assigns 1 to Bet but 0 to Land), the indicative conditional (2a) is evaluated as false, as desired.

For (3a), the model $M_3 = \langle U_3, V_3, F_3, A_3 \rangle$ in question is such that $U_3 = \{\text{Pressure}\}$, $V_3 = \{\text{Rainstorm}, \text{Sandstorm}, \text{Cloud}\}$, $F_3 = \{\text{Rainstorm} = \text{Pressure}, \text{Sandstorm} = 1 - \text{Pressure}, \text{Cloud} = \max(\{\text{Rainstorm}, \text{Sandstorm}\})\}$ and A_3 is such that $A_3(\text{Pressure}) = 1$, as indicated by Diagram 3-1.

Again, in order to evaluate the indicative conditional (3a), we go through the 3 steps mentioned above. After step 1, it is easy to see that there is only one extrapolated

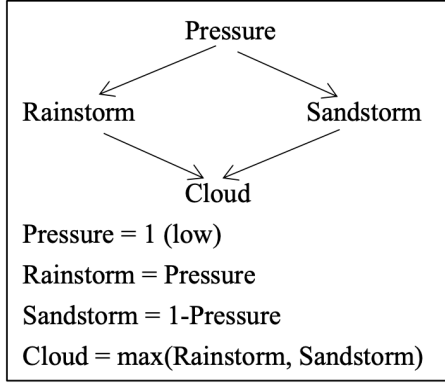


Diagram 3-1

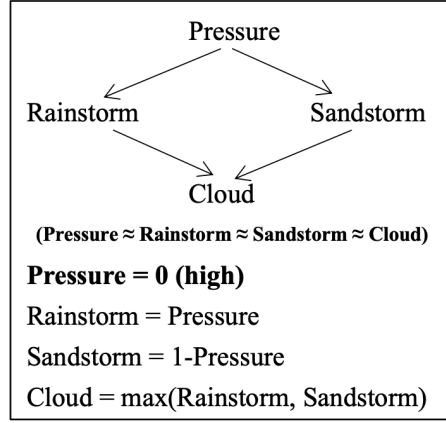


Diagram 3-2

model in $\mathbb{M}^{[\text{Rainstorm}=0]}$, as indicated by “Pressure = 0” in Diagram 3-2. Since the context presupposes nothing, so this extrapolated model is also in $\min(\mathbb{M}^{[\text{Rainstorm}=0]})$. Since the succedent of (3a) is true in this minimal model in $\mathbb{M}^{[\text{Rainstorm}=0]}$, the indicative conditional (3a) is evaluated as true, as desired.

3 Problems of Their Proposal and a Proposed Modification

So far, Deng & Lee’s account looks good. However, we will point out three problems of their account in this section and propose a simplified modification of it, so that, even if our final proposal in Section 4 won’t work out eventually, the modified version is to be preferred than theirs. Recall that the proposal of [3] requires us to go through three steps when evaluating the truth-value of an indicative conditional. We shall argue in this section that (i) the 1st step of their procedure has the problem that the complexity is high; (ii) the 2nd step of their procedure has the problems that (a) the ordering of extrapolated models via the priority relation provided by the context c will either be incorrect or be *ad hoc* and (b) the ordering is unnecessarily complicated (and even unnecessary); and (iii) the 3rd step of their procedure has the problem that it may give the wrong result for indicative conditionals with “impossible” antecedents. We will explain these problems one by one.

The first step in [3] asks us to use the model M and the antecedent φ to determine the extrapolated set $\mathbb{M}^{[\varphi]}$ of submodels, yet it is easy to see that, whenever there are n exogenous variables in the model M other than those mentioned or involved⁴ in the antecedent, there will be 2^n models in $\mathbb{M}^{[\varphi]}$ to be ordered by the priority relation. This problem can be illustrated by a simple example. Consider the following

⁴ An exogenous variable is involved in the antecedent iff it is an ancestor of some variable mentioned in the antecedent.

scenario: both Oswald and John were assigned the task to shoot Kennedy, though John somehow changed his mind when Kennedy's car was approaching. At the same time, Kennedy's two cooks, Mary and Jane, were preparing dinner for Kennedy. If Kennedy had not been killed, he would have had the dinner. In terms of causal model for this scenario, there will be 4 exogenous variables in this model: Oswald, John, MaryC (Mary cooks) and JaneC (Jane cooks), as indicated in Diagram 4.

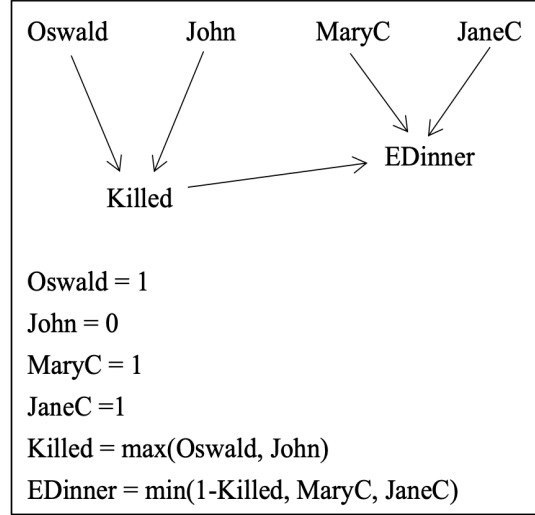


Diagram 4

Now, in order to evaluate an indicative conditional of the form “If Oswald did not kill Kennedy, ψ ” with respect to this scenario, Deng & Lee’s account requires us to consider every extrapolated model that makes the antecedent true. In this case, there are actually eight extrapolated models in $\mathbb{M}^{[\text{Oswald}=0]}$, differing only in assigning different values to the other three exogenous variables—John, MaryC, JaneC—that are not mentioned in the antecedent: $\langle 1, 1, 1 \rangle$, $\langle 1, 1, 0 \rangle$, $\langle 1, 0, 1 \rangle$, $\langle 1, 0, 0 \rangle$, $\langle 0, 1, 1 \rangle$, $\langle 0, 1, 0 \rangle$, $\langle 0, 0, 1 \rangle$, $\langle 0, 0, 0 \rangle$. From this example, one can easily see that, whenever there are n exogenous variables in a model other than those mentioned or involved in the antecedent, there will be 2^n models in $\mathbb{M}^{[\varphi]}$ to be ordered by the priority relation.

The problem of complexity involved in the 1st step may not impress logicians and philosophers, yet there are more serious problems in the 2nd step of the account of [3]. Once we have the set $\mathbb{M}^{[\varphi]}$ of extrapolated submodels, the next step is to use $\mathbb{M}^{[\varphi]}$ and the context c to determine the set of minimal extrapolated models relative to c via the priority relation provided by the context c . There are, however, two problems in this step: (a) the ordering of extrapolated models via the priority relation provided by the context c will either be incorrect or be *ad hoc* and (b) the ordering is unnecessarily complicated.

To see the problem (a), let us consider again the scenario considered in Diagram

4 and the indicative conditional “If Oswald did not kill Kennedy, Mary still cooked for Kennedy”. This indicative conditional seems to us to be intuitively true, especially when the investigation was still going on and the detective knew that the dinner had been prepared but unfortunately could not be enjoyed by Kennedy. Now, if we take all those eight extrapolated models mentioned in the previous paragraph to be minimal, then the conditional will be evaluated as false which is intuitively incorrect. So, in order to match with our intuition, the priority relation provided by c must order 4 of them—those that assign 0 to MaryC—as more distant than the other four. Yet, what reason(s) can one give for such an ordering? What can justify our assertion that the context c will provide such a priority relation? Nothing except that, otherwise, we won’t get our intuition right—but this reply is obviously *ad hoc*.⁵

To see the problem of (b) above, notice that the main purpose of the priority relation of a context c is to determine a set of “the most important truths in a model M ”, referred to as $\Delta_{M,c}$, which is supposed to be fixed or presupposed in c so that any extrapolated model that disagrees with $\Delta_{M,c}$ will be excluded as non-minimal w.r.t. M . If this is right, then, instead of ordering extrapolated models from M via ordering variables, we can simply take a context c to be a function that takes several factors in c as arguments and gives $\Delta_{M,c}$ as its value, and then use $\Delta_{M,c}$ to “filter out” those extrapolated models that are not minimal (see (IC*) below for details).

Finally, the 3rd step of applying (IC) to an indicative conditional has the problem that it may give the wrong result for conditionals with “impossible” antecedents w.r.t. a model M . A conditional has an impossible antecedent w.r.t. a model $M = \langle U, V, F, A \rangle$ iff its antecedent is not true no matter how one changes the value assignment A . A simple example is enough to explain this problem. Consider again the scenario described for Diagram 4 with two more added bits of information: (i) John was actually assigned as a backup assassinator for the event: if Oswald did not shoot, then John would, and (ii) the fact that Kennedy had a dinner cooked by Mary and Jane always make these cooks very happy, as indicated by Diagram 5 to the right. Now imagine someone who did not know that Kennedy was killed at that night but uttered (4) “If Kennedy ate dinner that night, Mary and Jane would be very unhappy”. It seems to us that this utterance is false rather than true. However, if we apply (IC) to this indicative conditional, it will be evaluated as true because $\mathbb{M}^{[EDinner=1]}$ is empty and the antecedent “Kennedy ate dinner that night” is impossible.

There is a simple way to avoid some of the problems mentioned above, especially the problem in the 3rd step and the problem (b) in the 2nd step; what is needed is only a small modification of (IC) in [3]. The ideas contained in the modification are simple: (i) we take an indicative conditional with an impossible antecedent w.r.t. a model M to behave in a similar way in M to a subjunctive conditional with the same

⁵In [3], Deng & Lee do not say much about the mechanism that determines the priority relation in a context c , and we don’t think that they can. Specifying the underlying mechanism in a context of conversation is probably the most notorious problem in logic and the philosophy of language.

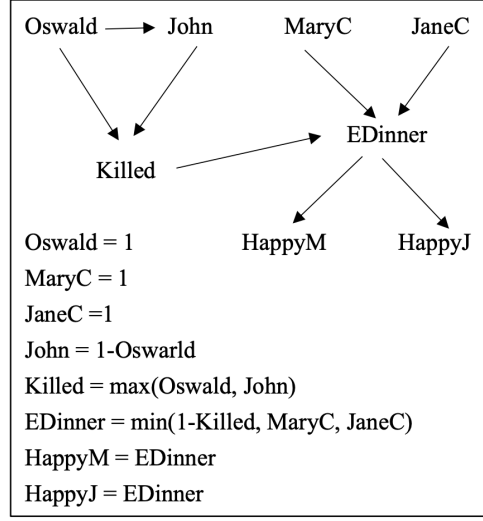


Diagram 5

antecedent and succedent. (ii) we take a context c to be a function from several factors in c to a set of truth $\Delta_{M,c}$ relative to M and take $\min(\mathbb{M}^{[\varphi]})$ to be the set of minimal extrapolated submodels “filtered in” by $\Delta_{M,c}$, i.e., the set of minimal models that make each member of $\Delta_{M,c}$ true. With these ideas, instead of (IC), i.e.,

- (IC) “ $\varphi \rightarrow \psi$ ” is true in a causal model M relative to a context c iff either (i) $\mathbb{M}^{[\varphi]}$ is empty, or (ii) “ ψ ” is true in all minimal extrapolated submodels in $\mathbb{M}^{[\varphi]}$ as ordered by the priority relation provided by c .

we take the following (IC*) to be a better stipulation for the truth-condition of an indicative conditional in a model M than Deng & Lee’s ([3]):

- (IC*) “ $\varphi \rightarrow \psi$ ” is true in a causal model M relative to a context c iff either (i) $\mathbb{M}^{[\varphi]}$ is empty and $\varphi > \psi$ is true in M , or (ii) $\mathbb{M}^{[\varphi]}$ is not empty and “ ψ ” is true in all minimal extrapolated submodels in $\mathbb{M}^{[\varphi]}$ that are filtered in by $\Delta_{M,c}$.

One can check that (4) are now false according to (IC*), as desired, while (1a)-(3a) still have the same truth-values as they have according to (IC).

4 A New Semantics for IC

Can (IC) and (IC*) be further improved? Can all the problems mentioned in the previous section be avoided at once? We think the answer is “Yes”, but it will take a more radical deviation from the basic ideas contained in [3]. We now explain how this can be done.

Let us start with a distinction that Pearl *et al.* ([10], Ch. 4, pp. 254–260) made

several times, though our distinction is a bit different from theirs⁶; the distinction is between a fully specified structural causal model (FSSCM) and a partially specified structural causal model (PSSCM). An FSSCM is a quadruple $\langle U, V, F, A \rangle$ satisfying the following three conditions that we mentioned before: (i) U and V are finite sets of variables; (ii) F is a set of structural equations; and (iii) A assigns values to each member of U , while a PSSCM is a similar quadruple $\langle U, V, F, A \rangle$ yet the function A in it is only a partial function, i.e., some members of U may not be defined. Intuitively, an FSSCM corresponds to a situation in which the subject has complete information about the values of all exogenous variables, while a PSSCM corresponds to a situation in which the subject has only incomplete information about the values of these variables. Pearl *et al.* ([9, 10]) remind us that PSSCMs are usually more realistic than FSSCMs in that researchers usually do not have full information about models that they are dealing with.

Why is this distinction important? Because, it seems to us, *a typical situation to utter an indicative conditional, unlike that to utter a counterfactual conditional, is when one has only partial or incomplete information about what are involved in a model.* To see this, one needs only to check again those cases that we mentioned in Section 2: (1a)-(3a) have their intuitive truth-values partly because the speaker does not know part of the facts involved in the model; especially, s/he lacks information about some exogenous variables.

The second thing we want to emphasize before making our proposal is the important role that a context plays when evaluating an indicative conditional. [3] briefly mentions that role and, we think, they are right to say that “[the] indicatives are epistemic in that they are about the world as we (now) know [or believe] it plus the supposition of its antecedent.” As to the question how our knowledge about the world in a context affects our evaluation of an indicative conditional, we think they are right again in saying that “in general, factors [in a context] such as conversational purposes, the subject’s beliefs, the shared presuppositions, etc., may all play a role in determining which [facts]⁷ are to be held fixed and which can be altered”.

In short, a context c has two roles to play when evaluating the utterance of an indicative conditional: first, the typical context for adequately uttering an indicative conditional is when the speaker does not have full information about the model in the context; second, factors in a context will determine a set of facts in the context to be held fixed or to be presupposed when evaluating an indicative conditional.

If what we have said so far in this section are correct, we propose that we should define “true-in-a-PSSCM relative to c ” for indicative conditionals first, then use it to define “true-in-a-FSSCM relative to c ”. For us, such an approach to indicative conditionals is to “treat an indicative conditional as an epistemic filter conditional”.

⁶They allow that a PSSCM may not have an equation for each endogenous variable.

⁷They actually use “variables” rather than “facts” in their paper, but the effect is the same: variable with assigned truth-values decides which simple facts are to be fixed.

(FCP) “ $\varphi \rightarrow \psi$ ” is true in a PSSCM $M = \langle U, V, F, A \rangle$ relative to a context c iff
 (i) “ φ ” is impossible in M and “ $\varphi > \psi$ ” is true in M , or (ii) “ ψ ” is possible
 in M and $Pr(\psi \mid \varphi, \Delta_{M,c}) = 1$ in M ⁸, where $\Delta_{M,c}$ is the set of simple facts
 about U that are held fixed in c .

A diagram with 'Oswald' and 'Someone' at the top, each with an arrow pointing down to 'Killed'.

Oswald

Someone

Killed

Diagram 6-1

Oswald	Someone	Killed
1	1	1
1	0	1
0	1	1
0	0	0

$\Pr(S = 1 \mid O = 0, K = 1) = 1$

Diagram 6-2

For (2a), it was reminded that the assertion of (2a) is intuitively false especially before the toss of the coin and before the bet. If that is so, then the proper situation for uttering (2a) is represented, not by Diagram 2-1, but by Diagram 7-1 below:

⁸Or, equivalently, ψ is true in every FSSCM M' that “extends” M and satisfies $\{\varphi\} \cup \Delta_{M,c}$, where a model M' extends M iff M' does not change anything in M but contains more “information” than M .

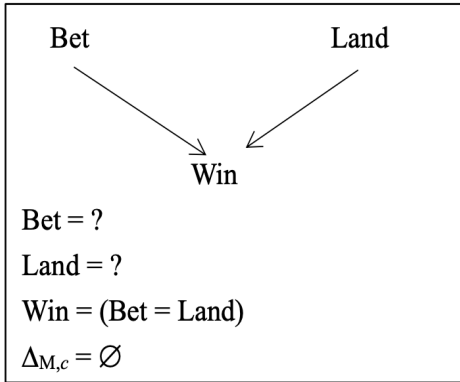


Diagram 7-1

Bet	Land	Win
1	1	1
1	0	0
0	1	0
0	0	1
$\Pr(W = 1 \mid B = 1, \emptyset) \neq 1$		

Diagram 7-2

and the presupposed set $\Delta_{M,c} = \emptyset$ as conditions to calculate the conditional probability of “ $W = 1$ ”. Since $\Pr(W = 1 \mid B = 1, \emptyset) \neq 1$ as indicated by Diagram 7-2 (assuming that $\Pr(B = 0, L = 1) \neq 0$; again, we filter out those lines that are incompatible with the conditions in Diagram 7-2), we reach the conclusion that (1a) is false by (FCP), as desired.

For (3a), it was reminded that the assertion of (3a) is intuitively true especially before you saw a rainstorm and therefore before you know the pressure. If that is so, then the proper situation for uttering (3a) is represented, not by Diagram 3-1, but by Diagram 8-1 below:

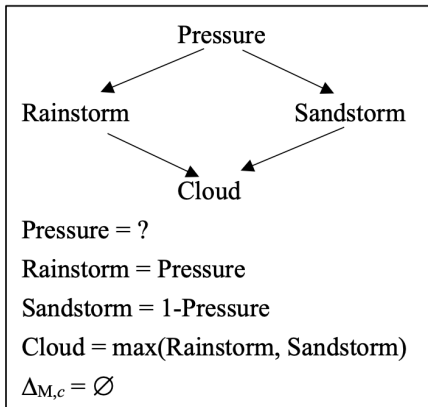


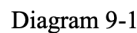
Diagram 8-1

Pressure	Rainstorm	Sandstorm	Cloud
1	1	0	1
0	0	1	1
$\Pr(\text{Cloud} = 1 \mid \text{Rainstorm} = 0, \emptyset) = 1$			

Diagram 8-2

Now, since the antecedent “A rainstorm isn’t approaching” of (3a) is possible in M , in order to decide the truth-value of (3a) relative to Diagram 8-1 by (FCP), we use the antecedent and the presupposed set $\Delta_{M,c} = \emptyset$ as conditions to calculate the con-

These examples show that our (FCP) judges (1a)-(3a) as having the same truth-values as judged by (IC) of [3], but they do not show us how the complexity problem mentioned in Section 3 is to be avoided. To see how (FCP) avoids the complexity problem, think again the scenario for Diagram 4: Oswald and John were suspected by the detective to be the unique assassin of Kennedy. At the same time, Kennedy's two cooks, Mary and Jane, were known by the detector detective to be preparing the dinner for Kennedy. If Kennedy had not been killed that night, he would have had the dinner. In this case, it is intuitively true for the detector detective to assert (5): "If Oswald did not Killed Kennedy, Mary would still cook for Kennedy". Clearly, the proper situation for uttering (5) is represented, not by Diagram 4, but by Diagram 9-1 below:

Diagram 9-2

Now, since the antecedent “Oswald did not kill Kennedy” of (5a) is possible in M , in order to decide the truth-value of (5a) relative to Diagram 9-1 by (FCP), we use the antecedent and the presupposed set $\Delta_{M,c} = \{\text{Killed} = 1\}$ as conditions to calculate the conditional probability of “MaryC = 1”. Since $Pr(\text{MaryC} = 1 \mid O = 0, K = 1) = 1$ as indicated by Diagram 9-2 (we filter out those lines that are incompatible with the conditions in Diagram 9-2), we reach the conclusion that (1a) is true by (FCP), as desired. In this case, we need only to consider four possibilities that are related to the detective’s ignorance, as shown in Diagram 9-2. If, however, we use (IC) of [3], we will, first of all, need to consider eight extrapolated models. This shows us how the complexity problems can in general be avoided: we just keep

intact those simple facts that we have already known.

So far, we have only defined the truth condition for an indicative conditional in a PSSCM relative to a context c . What if the model is an FSSCM? In order to define the truth-condition for an indicative conditional in an FSSCM relative to the same context c , let us first give the following definition:

Definition 1 (exogenous cause). $Y = y$ is an *exogenous cause* of $X = x$ in M iff (i) $Y \neq X$, (ii) Y is an exogenous variable in M , (iii) Y is an ancestor of X , and (iv) $A(Y = y) = 1$ in M .

Notice that, by (i) above, a fact about an exogenous variable in a model is not an exogenous cause of itself. We now call a PSSCM M' “the *truncated model* of an FSSCM M w.r.t. ‘ $\varphi \rightarrow \psi$ ’ relative to c ” iff M' is just like M except that M' has no information about the exogenous causes of $\Delta_{M,c}$ and the exogenous causes of variables in “ ψ ”. With these definitions, one can prove: (1) the model described in Diagram 6-1 is the truncated model of M_1 in Section 2 w.r.t. (1a) relative to c ; (2) the model described in Diagram 7-1 is the truncated model of M_2 in Section 2 w.r.t. (2a) relative to c ; (3) the model described in Diagram 8-1 is the truncated model of M_3 in Section 2 w.r.t. (3a) relative to c ; and (4) the model described in Diagram 9-1 is the truncated model of the model described in Diagram 4 in Section 3 w.r.t. (5) relative to c .

We now propose the following truth-condition (FCF) for an indicative conditional in a FSSCM relative to a context c :

(FCF) “ $\varphi \rightarrow \psi$ ” is true in an FSSCM $M = \langle U, V, F, A \rangle$ relative to a context c iff “ $\varphi \rightarrow \psi$ ” is true in the truncated PSSCM model of M w.r.t. “ $\varphi \rightarrow \psi$ ” relative to c .

The idea of (FCF) is simple: in case the utterance of an indicative conditional is suitable with full information about a model, not only does the context c will “hold fixed” some known facts in the model, it will also suggest that some known exogenous causes, i.e., those of $\Delta_{M,c}$ and the succedent of the conditional, should be ignored. With (FCF) and the four claims in the last paragraph, one can easily prove that: (1a)–(5) have the desired truth-value in the same context c no matter in an FSSCM or in a PSSCM.

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认知过滤式指示条件句

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摘 要

Deng 和 Lee (2021) 使用“外推式”因果模型语义学对指示条件句进行了刻画。这种刻画方式要求我们对变量先进行优先级排序并用它计算出外推子模型的排序, 然后在此基础上定义出最小外推子模型集从而解释指示条件句的语义。在本文中, 我们首先论证这一刻画指示条件句的方式会产生不必要的复杂性, 并且可能会产生错误的结果。随后, 我们提出了一种对指示性条件句的“认知过滤式”刻画, 并证明这种刻画可以成功地避免上述问题。

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