A Similarity-Based Signaling Game*

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Abstract. Language use and pragmatic inference frequently involve similarity reasoning. Based on the classical Lewis's signaling game, we propose a novel signaling game based on the similarity reasoning and the propositional contents of the signals. Within the new similarity-based signaling game, we discuss that the new game can select the best signals even when the available signals are less structured than what we want to communicate. Moreover, we apply our model to Horn's analysis of division of labors in Pragmatics.

1 Introduction

Language learning and language use frequently involve analogical reasoning. Analogical reasoning is fundamental to human thoughts. It relies on the comparison of two objects, or two systems, then highlights the similarity between them and makes prediction.([2]) We call it similarity reasoning in this paper. According to Gardenfors's concept space ([6]), concept formation relies on a similarity comparison between the features of objects. Meanwhile, similarity reasoning is also widely applied in the word-sense disambiguation in machine learning ([1, 12]) and game theoretical pragmatics ([14, 11]). Hence, our new signaling game emphasizes this similarity reasoning in the language use.

As we consider natural language, what can be said is often less than what we want to communicate. According to the principle of least effort ([21]), the language is always not fruitful enough to accurately express everything we want to express. Most of the time, we have to choose the closest word from the available ones to express our ideas. The process involves the comparison between the available words and the trade-off between the available words and the information we want to transmit.

Signaling game pioneered by David Lewis ([13]) has been applied in the study of various language features and language learning in philosophy, game theory and logic. The study of language includes the study of ambiguity ([16]), vagueness ([15, 14]), compositionality ([5]). The exploration about language learning addresses learning in social network ([22, 20]), learning communication contexts ([19, 3]) and learning concept structures ([11]).

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The traditional Lewis signaling game captures a rather simple communication scenario. In this model, there is a finite set of states. For each state, there is an appropriate action for that state. There is a sender, who is aware of the state and can send one signal to a receiver. The receiver is unaware of the state and must choose an action. The receiver can condition her choice of actions on the signal sent by the sender. Both the sender and the receiver have common interest, they would like the receiver to take the appropriate action given the state.

Within this model, the states, the signals and the acts are all assumed to be abstract objects. There is no content and structure within them. Because of this feature, there are several disadvantages of applying this model in the study of natural language. Firstly, we cannot talk about the meanings of the signals within the model. The meanings of the signals are only implicitly expressed by the equilibrium strategies of the game. Moreover, without the pre-assumed meanings of the signals, there are many equilibria in this game, some of which are inferior than others with respect to the outcomes. Secondly, we cannot talk about the connections between the states and the signals or between the signals and the acts within the model, since they are all abstract symbols. Lastly, the Lewis's signaling game does not allow us to discuss about the differences of the signals within the model. This is so because the format of signals in the model has no effect on the utilities.

To overcome these disadvantages, we propose a new signaling game based upon propositional logic. The states, signals and acts are all represented by propositional formulas. In addition, the format of the signals affects the players' utilities. There are two main advantages of doing this.

Firstly, the new model can discuss more complex messages by using propositional formulas. The expressiveness of the model is significantly improved. A similar idea has already been mentioned in [17, 18]. Meanwhile, the number of separating equilibria can also be reduced by considering the different meanings of the messages.

Secondly, using propositional logic and the entailment measurement between formulas from Dalal ([4]) and Hunter ([10]), we can discuss the similarity connections between the states and the signals and between the signals and the acts. As a result, the sender can choose a proper signal upon a specific state. That is, the sender chooses the signal by comparing the similarity between a given state and the signals, such that he can choose the most accurate expression. On the other hand, upon receiving the signal, the receiver chooses the most accurate explanation by comparing the similarity between the signals and the possible explanations. A successful communication requires the coordination between the states and the explanations through certain signals.

Moreover, the way we define the utility function in the new game is inspired by Horn's division of pragmatic labors.([9]) In a successful communication, a message needs to be sufficient (Q-principle: say as much as you can) as well as necessary (R-principle: say no more then you need). Such an idea is reflected in the new utility

functions of the new game. We also discuss the possible connections between our game and Horn's theory.

The rest of the paper is organized in the following way. Section 2 is a review of Lewis signaling game. In Section 3, we introduce the propositional logic and the model entailment measurement that we shall use in the later sections. Section 4 introduces our new signaling game based on the propositional logic and the similarity reasoning. In Section 5, we connect our model with Horn's division of pragmatic labors. The paper ends with a short conclusion.

2   Review of Signaling Game

The traditional Lewis's signaling game ([13]) provides a baseline model to capture a rather simple communication scenario. In this model, there is a finite set of states. For each state, there is an appropriate action for that state. There is a sender, who is aware of the state and can send one of a finite set of signals to a receiver. The receiver is unaware of the state and must choose an action. The receiver can condition her choice of actions on the signal sent by the sender. Both the sender and the receiver have common interest, they would like the receiver to take the appropriate action given the state.

Formally, a signaling game can be defined as follows.

Definition 1.   A signaling game G is a tuple $\{T, M, A, \{S, R\}, \sigma, U\}$, in which,

- T is a finite set of states;
- M is a finite set of signals;
- A is a finite set of actions;
- There are two players: a sender (S) and a receiver (R);
- The sender's action is $m \in M$ and the receiver's action is $a \in A$ [1];
- σ is a common prior on T;
- We assume that the players share the same utility, that is

$$U(t_i, m, a_j) = \sum_{t_i \in T} \sigma_R(t_i | m(t_i)) u(t_i, a_j), t_i \in T, a_j \in A, m \in M;$$

   in which

$$u(t_i, a_j) = \left\{ \begin{array}{ll} 1, & \text{if } i = j; \\ 0, & \text{otherwise.} \end{array} \right.$$

$\sigma_R(t_i | m(t_i))$ represents the receiver's conditional beliefs about the states after observing the sender's message $m(t_i)$. $u(t_i, a_i)$ measures the information matching between the states and the acts. If the state and the act matches, then the payoff is 1.

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[1]Whenever there is no confusion, we use the message and action symbols m and a as the players' strategy function as well, that is $m : T \to M$ and $a : M \to A$.

Otherwise, it is 0. The players' utility is the expected payoffs based on the receiver's beliefs about the states.

From the game theoretical point of view ([7]), signaling game is a typical example of Bayesian dynamic game with incomplete information. Following the standard analysis of the equilibrium, we specify the players' beliefs and their best response strategies as follows.

Definition 2.   (1) After observing the message $m_j \in M$ from the sender, the receiver has a belief about the type of the sender. This belief can be expressed by the probability $\sigma(t_i|m_j)$, such that $\sigma(t_i|m_j) \geq 0$ for any $t_i \in T$ and $\sum_{t_i \in T} \sigma(t_i|m_j) = 1$.

(2) Given such a belief, the receiver's best strategy $a^*(m_j)$ is to solve the following question,

$$argmax_{a_k \in A}\sigma(t_i|m_j)U(t_i, m_j, a_k).$$

(3) For each state $t_i \in T$, the sender's optimal strategy $m^*(t_i)$ solves

$$argmax_{m_j \in M}U(t_i, m_j, a^*(m_j)).$$

(4) $\sigma(t_i|m_j)$ follows from the Bayes's rule.

Definition 3.   A pure-strategy Bayesian equilibrium in the signaling game G is a pair of strategies (m^*, a^*) and a belief $\sigma(t_i|m_j)$ satisfying (1) through (4) in Definition 2.

There are two kinds of equilibria in signaling games, namely, pooling equilibrium and separating equilibrium. We use the following example to illustrate these two notions.

Example 1.   The simplest Lewis's signaling game consists of two states: $\{t_1, t_2\}$, two signals $\{m_1, m_2\}$ and two acts $\{a_1, a_2\}$.

An instance of "separating equilibrium" in this game has the following pattern

$$t_1 \longrightarrow m_1 \longrightarrow a_1$$

$$t_2 \longrightarrow m_2 \longrightarrow a_2$$

When state t_1 occurs, the sender sends m_1. By receiving m_1, the receiver takes a_1. Similarly, when state t_2 occurs, the sender sends m_2. Under this equilibrium, all the information about the states is communicated successfully between the sender and the receiver. Meanwhile, the players receive the highest payoff.

In addition, the game has a "pooling equilibrium". The following is the pooling equilibrium of this example.

$$t_1 \longrightarrow m_1 \longrightarrow a_1$$
$$t_2 \qquad m_2 \qquad a_2$$
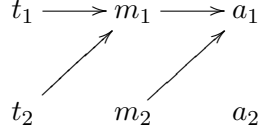
In this equilibrium, the sender always sends the same signal and the receiver takes the same action by ignoring the signal. As a result, the information is not fully communicated, pooling equilibrium of this kind features ambiguity, m_1 means both state t_1 and state t_2. Furthermore, these equilibria are Nash equilibria and are therefore stable in a certain sense. Neither the sender nor the receiver can unilaterally improve their situation by switching their strategies.

Though Lewis's signaling game provides a general characterization of communication and a theory of language meaning, the language in this model has no internal structures. In classical signaling games, the states, the signals and the actions are abstract objects. There is no structures within the states and the actions. The signals have no pre-assumed meanings. One consequence of these assumptions is that there are many equilibria in this game. For instance, if we switch m_1 and m_2 in the above mentioned separating equilibrium, it is still a separating equilibrium.

In this paper, we construct a signaling game based upon propositional logic such that we can talk about the propositional contents about the states, acts and the signals. Moreover, we can define the relationship between the states, the signals and the actions by using the properties in propositional logic. To achieve this goal, we need some basic notions in propositional logic. In the next section, we give a brief introduction of the propositional logic as well as the similarity reasoning that we will use in our model.

3   Similarity Reasoning in Propositional Logic

The similarity reasoning applied in this paper follows from Hunter's discussion about the entailment relationship between formulas.([10]) A similar idea can trace back to Dalal's work on belief revision.([4]) Since the entailment relationship measures how common two formulas (or sets of formulas) are with respect to their models, we call it similarity measurement in our paper.

Throughout the paper, we use $p, q, h. \ldots$ to represent atomic propositions, $\alpha, \beta, \gamma, \ldots$ to indicate the propositional formulas and $X, Y, \ldots,$ to denote the set of formulas. Deduction in classical propositional logic is denoted by the symbol $\vdash$.

Definition 4.   A language of the propositional logic $\mathcal{L}$ consists of the following.

(1) A finite set of atomic propositions;
(2) A set of connectives $\{\wedge, \vee, \neg, \rightarrow\}$.

Since we want to express various expressions in natural language, therefore, we use all of the connectives in the model. The well formed formulas based on $\mathcal{L}$ is defined in a standard way as in propositional logic.

Definition 5. Given a finite set of atomic propositions A, a model w of a formula ϕ (or a set of formulas X) is a subset of A, such that $\forall p \in w$, p is assigned true and $\forall p \in A \backslash w$ is assigned false. Let $I(X) = \mathcal{P}(atom(X))$ [2] be all the models of X delineated by the atoms used in X, where $atom(X)$ represents all the atoms in X.

Now we can define the entailment measurement between sets of formulas based on the models of propositional formulas.

Definition 6. Let $M(X, Y)$ be the models of X in $I(Y)$. Formally, $M(X, Y) = \{w \in I(Y) \mid w \models \bigwedge X\}$, where $\models$ is the logical entailment in propositional logic, $\bigwedge X$ represents the conjunction of all the formulas in X. Then, **the degree of entailment** of X for Y, denoted as $E(X, Y)$ is defined as follows:

$$E(X, Y) = \frac{|M(X, X \cup Y) \cap M(Y, X \cup Y)|}{|M(X, X \cup Y)|}$$

in which, given any set X, $|X|$ represents the number of elements in this set.

Example 2. Suppose $X = \{p\}, Y = \{p \vee q\}$. Thus, $I(X \cup Y) = \{\{p\}, \{q\}, \{p, q\}, \emptyset\}$, $M(X, X \cup Y) = \{\{p\}, \{p, q\}\}$ and $M(Y, X \cup Y) = \{\{p\}, \{q\}, \{p, q\}\}$. Therefore, $E(X, Y) = 1$, while $E(Y, X) = 2/3$.

We call the degree of entailment between two sets as similarity measurement. The similarity measurement has the following properties.

Proposition 1. *Let X, Y and Z be the sets of classical propositional formulas, then the followings hold*:

(1) $0 \leqslant E(X, Y) \leqslant 1$;
(2) $X \vdash \wedge Y$ *iff* $E(X, Y) = 1$;
(3) $X \vdash \neg \wedge Y$ *iff* $E(X, Y) = 0$.

Proposition 1 shows that there is a relationship between the similarity measurement and the logical entailment. Because of this feature, we can connect this measurement to Horn's pragmatic principles in conversation in later sections.

It is also obvious that the similarity measurement is not symmetric. That is, given any X, Y, $E(X, Y)$ is usually not equal to $E(Y, X)$.

Definition 7. We represent a propositional logic with a similarity measurement denoted by SL as a pair $SL = (L, E)$ in which

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[2] $\mathcal{P}(X)$ represents the power set of X.

(1) L is a propositional logic defined on $\mathcal{L}$;

(2) $E(X, Y)$ is a similarity measurement of X for Y, in which X, Y are formulas (or sets of formulas) in L.

4 A Similarity-Based Signaling Game

Two reasons motivates us to consider a similarity-based signaling game. Firstly, analogical reasoning appears very frequently in language learning and language use, which is fundamental to human thoughts. As Gardenfors stated in his theory of concept space ([6]), concept formation also relies on a similarity comparison between the features of objects.

Secondly, regarding natural language, the language is always not fruitful enough to accurately express everything we want to express. In addition, there are numerous uncertainties in language, such as ambiguity, vagueness and synonymy. Hence, what we can say is often much less accurate than what we want to communicate. As a result, we have to choose the most appropriate word from the available ones to express our ideas. During this decision process, it is often observed the comparisons between the available words and the trade-offs between the available words and the information we want to transmit.

On the other hand, in Lewis's signaling game, the signals have no effect on players' utilities. As a result, the model can not distinguish different forms of signals. Therefore, we need to modify the Lewis's signaling game such that we can compare between different signals with respect to the contents of the signals. In addition, the contents of the signals should affect the utilities.

Therefore, in this section, we define a similarity-based signaling game. Comparing to the signaling game defined in Section 2, there are two main changes we made in the new game.

Firstly, we define the states, the signals and the actions in the standard signaling game with propositional contents. Formally, we use propositional formulas representing the states, the signals and the actions in the signaling game.

Secondly, we redefine the utility function in the signaling game such that the similarity relations between the states and the signals, and between the signals and the actions can be reflected in the utility function. By using the similarity measurement E defined in Definition 6, we emphasize the similarity comparison of propositional contents in communication. As a result, the choice of the signals affects players' utilities as well as the outcome of the game.

These two changes are included in the new definition of our game presented below.

Definition 8.   A similarity-based signaling game SG is a tuple
$$SG = \{SL, T, M, A, \{S, R\}, \sigma, U\}, \text{ in which}$$

- SL is a propositional logic with a similarity measurement E;
- T is a finite set of states, $t \in T$ is a propositional formula in SL;
- M is a set of signals, $m \in M$ is a propositional formula in SL;
- A is a finite set of actions, $a \in A$ is a propositional formula in SL;
- There are two players: a sender (S) and a receiver (R);
- σ is a uniform prior distribution over T, σ is the common knowledge between the players [3];
- The sender's action set is M and the receiver's action set is A;
- We assume that the players share the same utility, that is

$$U(t_i, m, a_j) = \sum_{t_i \in T} \sigma_R(t_i \,|\, m(t_i))\{u(t_i, a_j) + E(m(t_i), t_i) \times E(a_j, m(t_i))\}$$

in which

$$u(t_i, a_j) = \begin{cases} 1, & \text{if } t_i \text{ and } a_j \text{ are logically equivalent;} \\ 0, & \text{otherwise.} \end{cases}$$

The utility function consists of two parts. The first part $u(t_i, a_j)$ characterizes the information matching between the states and the acts. If the states and the acts are logically equivalent, then we say the information transaction is successful.

The second part $E(m(t_i), t_i) \times E(a_j, m(t_i))$ in the utility function represents the similarity between the states and the signals and between the signals and the acts with respect to their propositional contents. We can calculate the similarity measure of $m(t_i)$ for t_i, with $E(m(t_i), t_i)$, and the similarity measure of a_j for $m(t_i)$ by $E(a_j, m(t_i))$. The reason we consider the multiplication of these two measures is because $E(m, t) \leq 1$ and $E(a, m) \leq 1$, therefore $E(m, t) \times E(a, m) \leq 1$, which yields the same maximum as $u(t, a)$. Hence, the two parts in the utility function have the same impact on the overall utility.

The way how we define the similar measurement is inspired by Horn's devision of pragmatic labors ([9]). According to Horn, Grice's maxims can be replaced with two fundamental principles: say as much as you can (the Q-principle) and say no more than you must (the R-principle).([9, 8]) The Q-principle states a sufficient condition about the quantity of the message, while the R-principle states a necessary condition about the relevance of messages.

On the other hand, our similarity measurement has a close connection with the logic entailment. According to Proposition 1, given a set of formulas X and Y, $E(X, Y) = 1$ if and only if $X \vdash \wedge Y$. Thus, X represents the sufficient condition of Y. Therefore, we define that the sender is to choose a message implying the state

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[3]Throughout the paper, we assume σ is a uniform distribution over T. It is easy to extend the model for other prior distributions.

given the state t as $E(m, t)$. Also, from the receiver's point of view, the message provides a necessary condition for his interpretation of the message, namely, $E(a, m)$. Regarding the connection between our game and Horn's theory, we will discuss it more in Section 5.

In the new game, the states, the signals and the acts are all represented by propositional formulas. Hence, we can express the meaning of the signals as the propositional contents. The acts in the form of propositional formulas can be understood as the receiver's interpretations of the signals.

The definition of Bayesian equilibrium for SG is defined in the same way as in Definition 3.

Definition 9. A pure-strategy Bayesian equilibrium in the signaling game SG is a pair of strategies (m^*, a^*) and a belief $\sigma_R(t_i|m_j)$ satisfying (1) through (4) in Definition 2.

We use the following examples to illustrate the notions in the SG game.

Example 3. Given a SG game, suppose the set of the states is $\{t_1: p \wedge q, t_2: p \vee q\}$, the set of signals is $\{m_1: p \wedge q, m_2: p \vee q\}$, and the set of the acts is $\{a_1: p \wedge q, a_2: p \vee q\}$

Obviously, the equilibrium strategy is $m(t_1) = m_1$, $m(t_2) = m_2$, $a(m_1) = a_1$ and $a(m_2) = a_2$. That is

$$t_1: p \wedge q \longrightarrow m_1: p \wedge q \longrightarrow a_1: p \wedge q$$

$$t_2: p \vee q \longrightarrow m_2: p \vee q \longrightarrow a_2: p \vee q$$

This is a separating equilibrium under which the information is completely transferred from the sender to the receiver. Given the prior σ as a uniform distribution over T, the utility under this equilibrium is

$$\begin{aligned} U =& \frac{1}{2}(u(t_1, a_1) + E(m_1, t_1) \times E(a_1, m_1)) + \\ & \frac{1}{2}(u(t_2, a_2) + E(m_2, t_2) \times E(a_2, m_2)) \\ =& 2 \end{aligned}$$

It is worth commenting more on this simple example and emphasizing the differences between the new SG game and the classical Lewis's signaling game.

(1) Since the propositional contents of signals are specified in SG, the number of separating equilibria is reduced to only one in Example 3. While in Example 1 with the similar setting, there are two different separating equilibria under which the signals switch meanings in the two equilibria.

(2) Example 3 is a special case that the signals take exactly the same form as the states. Normally, the signals may have different forms as the states. Example 4 illustrates this point.

(3) It is even possible that the set of the states does not need to be exactly the same as the set of the acts. As a result, the element $u(t, a)$ may take values rather than just 0 or 1. We can measure the similarity of t and a by calculating $E(t, a)$ and $E(a, t)$. We leave this point as future work.

As we mentioned before, natural language is often not exactly the same as the information we want to communicate. Therefore, the situation in Example 3 hardly occurs in natural language. Hence, we have to consider the situation where the set of signals are different from the set of the states, which is characterized in Example 4.

Example 4.   Given a SG game, suppose the set of states is $\{t_1\colon p, t_2\colon p \vee q\}$, the set of signals is $\{m_1\colon p, m_2\colon q\}$, and the set of acts is $\{a_1\colon p, a_2\colon p \vee q\}$.

We calculate the E measure for each state, signal and act pairs as follows.

$$E(m_1, t_1) = 1, E(m_2, t_1) = \frac{1}{2}, E(m_1, t_2) = 1, E(m_2, t_2) = 1;$$

$$E(a_1, m_1) = 1, E(a_2, m_1) = \frac{2}{3}, E(a_1, m_2) = \frac{1}{2}, E(a_2, m_2) = \frac{2}{3}.$$
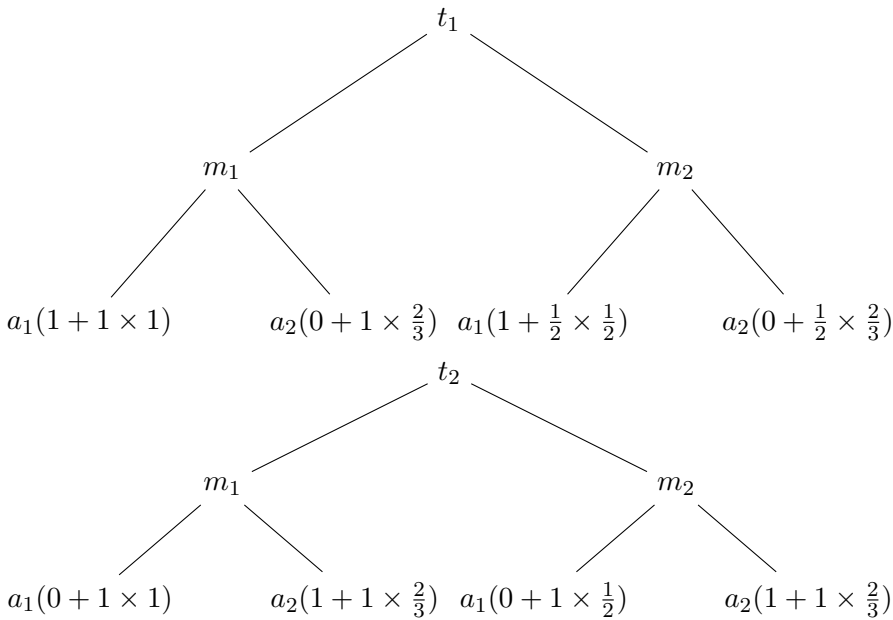

Figure 1: Extensive play for each state (Example 4)

In Figure 1, we represent the play of the game as an extensive game for each

state. The sender knows the true state while the receiver does not know. And $\sigma(t_1) = \sigma(t_2) = 0.5$. The number associated with each act is the payoff for the players.

The equilibrium of this game can be found in this way. For the receiver, once m_1 is received, he decides the action by calculating the expected utility of choosing each act. Since he does not know which state is actually occurs, he calculates the expected utility of choosing a_1 with respect to the posterior probability on the states, which we assume them to be equally likely. Therefore, $EU(a_1) = \frac{1}{2} * 2 + \frac{1}{2} * 1 = \frac{3}{2}$. The expected utility of choosing a_2 is $EU(a_2) = \frac{1}{2} * \frac{2}{3} + \frac{1}{2} * \frac{5}{3} = \frac{7}{6}$. Hence, he should choose a_1 for m_1. In the similar way, we can calculate that the receiver chooses a_2 for m_2. By assuming the receiver's strategy, the sender's best strategy is to send m_1 for t_1 and m_2 for t_2.

Therefore, there is a separating equilibrium for this game that is $m(t_1) = m_1$, $m(t_2) = m_2$, $a(m_1) = a_1$, $a(m_2) = a_2$.

Example 4 shows that our model can induce a successful communication even if the set of the signals is not exactly the same as the set of the states. At the same time, the equilibrium sustains our intuition about the propositional contents of the signals.

From Example 4, it is obvious that when the communication is successful, the similarity measurement $E(m, t) \times E(a, m)$ also achieves the optimal. This result can be generalized.

Proposition 2. *Given a SG game, if the separating equilibrium exists, then the message m used in the equilibrium is the most similar signal, that is, m maximize $\sum_{t \in T} E(m, t) \times E(a, m)$.*

Proof.   When the separating equilibrium exists, then $t = a$ (or logically equivalent). Suppose m is the message used in this separating equilibrium. Then, for any other message m' that is not logically equivalent to m, $EU(t, m, a) \geqslant EU(t, m', a)$. Hence, $\sum_{t \in T} u(s, t) + E(m, t) \times E(a, m) \geqslant \sum_{t \in T} u(s, t) + E(m', t) \times E(a, m)'$. Therefore, $\sum_{t \in T} E(m, t) \times E(a, m) \geqslant \sum_{t \in T} E(m', t) \times E(a, m')$.   □

Proposition 2 shows that our new signaling game can select the best signals for a successful communication in terms of our similarity measurement.

5   Horn's Division of Pragmatic Labors

In game-theoretical pragmatics, the sender must take into account how the receiver would interpret the signals when he chooses the signal. Meanwhile, the sender should consider the available set of signals and the fact that the sender chooses the signal strategically.

On the other hand, the similarity measurement defined in our game not only measures the common part of two propositions, but also implies the entailment relationship between two propositions. Recall the definition of the similarity measurement

in Definition 6 and the features of the measurement in Proposition 1, the following holds.

Given a set of propositional formulas X, and Y, $E(X, Y) = 1$ if and only if $X \vdash \wedge Y$. By using this property, we can discuss about the Horn's division of pragmatic labors, then connect the equilibrium strategy in our new signaling game to the pragmatic principles in Horn's theory.

As we mentioned before, according to Horn's division of pragmatic labors, Grice's maxims can be replaced with two fundamental principles: say as much as you can (the Q-principle) and say no more than you must (the R-principle).([9, 8]) These two principles can be characterized by our similarity reasoning as follows.

Definition 10. In a two-player signaling game SG, the Q-principle corresponds to the sender's strategy (the message implies the state information, i.e., the message should be sufficient):

$$s^* = argmax_{m \in M} E(m, t), t \in T$$

and the R-principle corresponds to the receiver's strategy (the interpretation should imply the message, i.e., the message should be necessary):

$$r^* = argmax_{a \in A} E(a, m(t)), m(t) \in S^*$$

We call the strategy pair (s^*, r^*) as the max-pragmatic strategy.

In general, the max-pragmatic strategy is not the Bayesian equilibrium of SG, because the pair of the numbers that maximizes with respect to A and B separately does not equal the pair that maximizes with respect to $A \times B$. The following proposition provides the sufficient condition for the connection between these two concepts.

Proposition 3. $(m^*, a^*) = (s^*, r^*)$ *if the following holds. For each* $t_i \in T$,

$$argmax_{(m,a)}(u(t_i, a_j) + E(m(t_i), t_i) \times E(a_j, m(t_i))) =$$

$$\{argmax_{m \in M} E(m(t_i), t_i) \otimes argmax_{a \in A} E(a, m(t_i))\}$$

in which $\otimes$ *represents the Cartesian product.*

Intuitively, if the players' strategies maximizing the similarity measure with respect to the state, signal pair and the signal, act pair separately are the same strategies that maximize the utility function in SG, then the max-paragamatic strategy is the same as the Bayesian equilibrium strategy in the SG game.

Conceptually, Proposition 3 shows the potential applications of our SG game in pragmatics. It provides a pragmatic explanation for the Bayesian equilibrium in SG game. Moreover, in contrast with the Bayesian Equilibrium, the max-pragmatic

strategy is easier to calculate. Thus, Proposition 3 provides a simple way to test and find the Bayesian Equilibrium through the max-pragmatic strategy.

6   Discussion and Conclusion

By constructing the signaling game with propositional logic, we demonstrate that the signals in the game can have pre-assumed propositional contents. By considering a new utility function, the players in the game not only consider the outcome of the information transactions but also take into account the language in use. The choice of the set of signals may affect the payoff of the game as well as the optimal strategies of the game.

In addition, the game has the potential to be extended to various directions. We mention two possibilities here.

1. **Dynamic context and dynamic meaning**

   The context of communication and meaning of words change all the time. It would be useful if the current model can deal with the revision of the content of a context and meaning of words. We use the revision operator from Dalal's model on knowledge base revision ([4]), which follows two basic principles: (1) Primacy of new information: the revised knowledge should conform to the new information; (2) Persistence of old knowledge: as much old knowledge as possible should be retained in the revised knowledge, which means the revised knowledge base is obtained by some form of minimal change.

   The basic idea of the revision operator $\circ$ in $\alpha \circ \beta$ is first to extend the models of α in $I(\alpha)$ by adding or deleting at most one atom for all the elements in $I(\alpha)$, then to take the intersection of the modified $I(\alpha)$ (denoted as $I^1(\alpha)$) with the new element $I(\beta)$. If there is no intersection, then repeat the process on $I^1(\alpha)$, until the intersection is not empty.

   For example, suppose $\alpha : \{p, \neg q\}$, $\beta : \{q\}$. Then $I(\alpha) = \{p\}$. $I^1(\alpha) = \{\emptyset, \{p\}, \{p, q\}\}$, $I(\beta) = \{\{q\}, \{p, q\}\}$. Thus, $I^1(\alpha) \cap I(\beta) = \{p, q\}$. Therefore, $\alpha \circ \beta$ is the formula where its only model is $\{p, q\}$.

   Applying this revision operation, we can calculate the similarity relation between the revised new formula and the context. For instance, we can calculate $E(T, \alpha \circ \beta)$ when the meaning of the word changes or $E(T \circ \beta, \alpha)$ when the state of the world changes.

2. **Negation and conflicts**

   As we know when there is a conflict for any X and Y, $E(X, Y) = 0$. Thus, the similarity measurement alone is not enough to capture the intuition in language communication. A conflict measurement can be combined with the similarity measurement to deal with the degree of conflicts between formulas. Such

a measurement is already stated in the work in [10]. The degree of conflict $C(X, Y)$ between X and Y is defined as:

$$C(X, Y) = \frac{\min(Distance(X, Y))}{\log_2(|I(X \cup Y)|)}$$

In this formula, $Distance(X, Y)$ represents all the hamming distance between the models of X and Y, and is defined in the following way.

Let X be a set of formulas and let $w_i, w_j \in I(X)$. Then $Distance(X, Y) = \{Dalal(w_x, w_y) \mid w_x \in M(X, X \cup Y) \text{ and } w_y \in M(Y, X \cup Y)\}$ where $Dalal(w_i, w_j) = |w_i - w_j| + |w_j - w_i|$. The Dalal distance between w_i and w_j, denoted $Dalal(w_i, w_j)$, is the difference in the number of atoms assigned true.

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基于相似性推理的信号博弈

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摘　　要

　　语言使用和语用推理经常涉及相似性推理。基于经典的刘易斯（D. Lewis）信号博弈，我们提出了一种基于相似性推理和信号命题内容的信号博弈。在新的基于相似性推理的信号博弈中，我们讨论了即使已有的信号表达力不如我们想要传达的信息，该模型仍旧能够在已有的信号中选出最优的信号。此外，我们还将该模型应用于霍恩（L. Horn）对于语用学分工的分析。

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