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Characterizing Argumentation Frameworks with an Extension*

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Abstract. According to a given criterium, from the structure of an argumentation framework, a set of extensions can be decided. Conversely, an extension or a set of extensions can identify a set of argumentation frameworks. The direction from argumentation frameworks to their semantics has been discussed a lot, but little attention has been paid to the opposite direction. In this paper, we focus on characterizing argumentation frameworks with a given set of arguments as an extension, and show its applications on the update of argumentation frameworks and monotony.

1 Introduction

Formal argumentation is a very active research area in the field of knowledge representation and reasoning, in which Dung's abstract argumentation ([11]) has been extensively studied in the past two and a half decades, including argumentation semantics ([20, 1]), algorithms ([15, 19]), computational complexity ([14, 13]), dynamics ([3, 7]), etc..

An abstract argumentation framework can be modeled as a graph (A, R), where A represents a set of arguments and $R \subseteq A \times A$ a binary relation called "attack". Given such a graph, an interesting question is which sets of arguments, i.e. extensions, can reasonably be accepted. This question is stated as semantics that is a function from an argument graph to a set of extensions. There is a rich variety of semantics, defined in terms of intuitions and principles ([1]), each of which represents a kind of attitude to select acceptable arguments in practice.

Example 1. AF_1 is an argumentation framework illustrated in Figure 1. Arguments a and c are accepted simultaneously under grounded semantics wich represents a caucious choice. Then $\{a, c\}$ is the grounded extension of AF_1. The semantics of AF_1 is a mapping from AF_1 to $\{\{a, c\}\}$.

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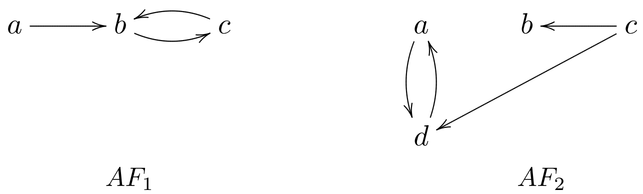


Figure 1: Argumentation frameworks

Conversely, from an extension or a set of extensions, a set of argumentation frameworks can be decided.

Example 2. Let $E_1 = \{a, c\}$ be a set of arguments. There are infinite argumentation frameworks that have E_1 as the grounded extension. All of them can be put in a set, denoted as $AF_{gr}^{E_1}$. AF_1 and AF_2 illustrated in Figure 1 are two argumentation frameworks in $AF_{gr}^{E_1}$.

The direction from argumentation frameworks to semantics has been discussed a lot ([20, 1, 15, 19]), but little attention has been paid to the opposite direction from semantics to frameworks. Example 2 gives rise to a research question: Given a set E of arguments, under grounded semantics, what kinds of conditions AF_{gr}^{E} should satisfy? It can be seen from the structures of both AF_1 and AF_2 that one argument in E is unattacked, and it can be seen more from the structure of AF_2 that the circle identified by $\{a, d\}$ is attacked by c in E. There should be a precise characterization of AF_{gr}^{E} that covers all of these conditions. We will discuss this problem in this paper, and will focus on characterizing the set of argumentation frameworks with an extension or a set of extensions.

In our previous works ([17]), we simplifed the computation of semantics of probabilistic argumentation by characterizing subgraphs. After that, we introduced this idea to "enforcement", one question of the dynamics of argumentation frameworks. Enforcement is to change an argumentation framework to make a set or sets of arguments accepted. Baumann et al. firstly investigated whether enforcing an extension is possible in [4]. We discussed the conditions under which an enforcement achieves ([22, 21]). In [22], we classified the change of an argumentation framework into two directions: expansion and contraction. The expansion of an argumentation framework is defined by adding arguments or attacks to the framework. The contraction of an argumentation framework is defined by deleting arguments or attacks from the framework. We formulated methods to expand or contract argumentation frameworks to enforce an extension under complete, grounded, preferred and stable semantics respectively. In [21], we discussed how to update an argumentation framewok to enforce an extension under complete, grounded, preferred and stable semantics, where updating an argumentation framework is not to change it in one direction, i.e., either expansion or contraction, but to form a new framework.

The way of characterizing subgraphs in [17] sparks an idea of characterizing argumentation frameworks from a fixed extension or a set of extensions. “Enforcement” in [22] and [21] provides methods to do the characterization of frameworks. This paper is motivated by these two points and to be an extension of [21] which incorporates the principle behind the enforcement in [21] and the idea of characterizing frameworks. It shows the relations between the structures of argumentation frameworks and some semantics, making a progress on the research direction from semantics to frameworks. Furthermore, it can be applied to the dynamics of argumentation frameworks which are about the interaction between the change of frameworks and that of semantics.

The structure of this paper is as follows. Section 2 introduces some basic notions of abstract argumentation. Section 3 is the main part of this paper, studying the characterization of argumentation frameworks with a given extension, and a given set of extensions. Section 4 shows the applications of our work to updating argumentation frameworks and monotony. Section 5 concludes.

2 Preliminaries

2.1 Argumentation frameworks

To make this paper self-contained, in this section, we introduce some basic notions on the abstract argumentation, including argumentation frameworks and their semantics. We consider only finite argumentation frameworks for the sake of simplicity, and all presentations here are adjusted to the studies in the following sections.

Definition 1. Let **U** be the universe of all possible arguments. An argumentation framework $\mathcal{G}$ is a tuple (A, R) where A is finite, $A \subseteq \mathbf{U}$ and $R \subseteq A \times A$ is a binary relation on A.

Let $B \subseteq A$ and $R_B = R \cap (B \times B)$. (B, R^*) is a sub-framework of $\mathcal{G}$ if $B \subseteq A$ and $R^* \subseteq R_B$. $\mathcal{G} \downarrow_B = (B, R_B)$ is the restriction of $\mathcal{G}$ to B, and it is a sub-framework of $\mathcal{G}$.

Given $a, b \in A$, $(a, b) \in R$ means a attacks b. We write aRb instead of $(a, b) \in R$, and $a\not{R}b$ instead of $(a, b) \notin R$. Given B, $C \subseteq A$, we say BRa (respectively, aRB) if there exists $b \in B$ such that bRa (respectively, aRb), and BRC if there exist $b \in B$ and $c \in C$ such that bRc.

We use $a\widetilde{R}b$ to denote the indirect relation between two arguments: $a\widetilde{R}b$ if there exist a series of arguments x_1, x_2, ... x_n such that aRx_1, x_1Rx_2, ..., and x_nRb.

Circles play a key role to dicide the content of an extention and the number of extensions of an argumentation framework. It is defined as follows.

Definition 2. Let $\mathcal{G} = (A, R)$ be an argumentation framework, and $B \subseteq A$. $\mathcal{G} \downarrow_B$ is a circle of $\mathcal{G}$ if and only if for any arguments $a, b \in B$, $a\widetilde{R_B}b$ and $b\widetilde{R_B}a$. The set of all circles of $\mathcal{G}$ is denoted as $CIR_{\mathcal{G}}$, and the set $\{B \mid \mathcal{G} \downarrow_B \text{ is a circle of } \mathcal{G}\}$ is denoted as $SCIR_{\mathcal{G}}$.

Example 3. Let AF_3 be an argumentation framework illustrated in Figure 2. $CIR_{AF_3} = \{AF_3 \downarrow_{\{a,b\}}, AF_3 \downarrow_{\{b\}}, AF_3 \downarrow_{\{c,d\}}\}$ and $SCIR_{AF_3} = \{\{a, b\}, \{b\}, \{c, d\}\}$.

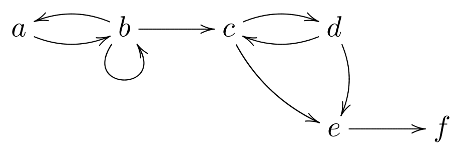


Figure 2: An argumentation framework

The notion of circle is different from that of strongly connected component of an argumentation framework in [2].

Definition 3. Let $\mathcal{G} = (A, R)$ be an argumentation framework, $PE_{\mathcal{G}}$ is a relation on A and satisfies:

- for any $x \in A$, $(x, x) \in PE_{\mathcal{G}}$;
- for any $x, y \in A$ with $x \neq y$, $(x, y) \in PE_{\mathcal{G}}$ if and only if $x\widetilde{R}y$ and $y\widetilde{R}x$.

$PE_{\mathcal{G}}$ is an equivalence relation and we call it the relation of path-equivalence. Let $a \in A$. The equivalence class of a modulo $PE_{\mathcal{G}}$ is a strongly connected component of $\mathcal{G}$. The set of strongly connected components of $\mathcal{G}$ is denoted as $SCCS_{\mathcal{G}}$ and it is a partition of A.

It can be seen from Definitions 2 and 3 that any cyclic graph of a strongly connected component is a circle, but not vice versa. Considering AF_3 in Example 3, $SCCS_{AF_3} = \{\{a, b\}, \{c, d\}, \{e\}, \{f\}\}$ in which two subframeworks induced by $\{a, b\}$ and $\{c, d\}$ depict circles of AF_3.

All circles in an argumentation framework make a contribution to the constitution of semantics, while the strongly connected components in a framework have some relations to the properties of semantics ([2]).

2.2 Semantics of argumentation

Given an argumentation framework, a fundamental problem is to determine which arguments can be regarded as collectively acceptable. There are mainly two approaches: extension-based approach and labelling-based approach. The idea underlying the extension-based approach is to identify sets of arguments, called extensions, that can be accepted according to a given criterion. The idea underlying the labelling-based approach is to assign a label to each argument according to a given criterion.

The extension-based approach starts from the notions of conflict-freeness and defense.

Definition 4. Let $\mathcal{G} = (A, R)$ be an argumentation framework, $a, b \in A$ and $E \subseteq A$.

- E is conflict-free if and only if for any $a, b \in E$, $a \not\mathcal{R} b$;
- E defends a if and only if for any bRa, ERb.

The set of all arguments defended by a subset of A can be denoted by the characteristic function of $\mathcal{G}$. The characteristic function makes a contribution to symplify the definitions in the extension-based approach.

Definition 5. The characteristic function of an argumentation framework $\mathcal{G} = (A, R)$ is $\mathcal{F} : 2^A \longrightarrow 2^A$, where for any $B \subseteq A$, $\mathcal{F}(B) = \{a \in A \mid B \text{ defends } a\}$.

Based on conflict-freeness and the characteristic function, a set of extensions can be defined as follows ([6, 9, 15]).

Definition 6. Let $\mathcal{G} = (A, R)$ be an argumentation framework, $a \in A$ and $E \subseteq A$.

- E is an admissible set if and only if E is conflict-free and $E \subseteq \mathcal{F}(E)$;
- E is a complete extension of $\mathcal{G}$ if and only if E is conflict-free and $E = \mathcal{F}(E)$;
- E is the grounded extension of $\mathcal{G}$ if and only if E is the minimal complete extension (with respect to set inclusion);
- E is a preferred extension of $\mathcal{G}$ if and only if E is a maximal admissible set (with respect to set inclusion);
- E is a stable extension of $\mathcal{G}$ if and only if E is admissible and $ER(A \setminus E)$;
- E is the ideal extension of $\mathcal{G}$ if and only if E is the maximal admissible extension (with respect to set inclusion) contained in all preferred extensions of $\mathcal{G}$.

The labelling-based approach is defined in terms of labellings. A labelling is a function assigning a label to each argument of an argumentation framework to indicate its status. There are usually three labels: **in**, **out** and **undec**. The label **in** indicates that the argument is accepted, **out** indicates that the argument is rejected and **undec** indicates that the argument is undecided which means that it can not be decided to be accepted or rejected ([1]).

Definition 7. Let $\mathcal{G} = (A, R)$ be an argumentation framework. The labelling of $\mathcal{G}$ is a total function $L : A \longmapsto \{\mathbf{in}, \mathbf{out}, \mathbf{undec}\}$.

Let $\mathbf{in}(L) = \{a \in A \mid L(a) = \mathbf{in}\}$, $\mathbf{out}(L) = \{a \in A \mid L(a) = \mathbf{out}\}$ and $\mathbf{undec}(L) = \{a \in A \mid L(a) = \mathbf{undec}\}$. L is often represented as a triple

$(\mathbf{in}(L), \mathbf{out}(L), \mathbf{undec}(L))$.

Let $B \subseteq A$. $L \downarrow_B = (\mathbf{in}(L) \cap B, \mathbf{out}(L) \cap B, \mathbf{undec}(L) \cap B)$ is called the restriction of L to B.

The central criterion for labelling-based approach is legality.

Definition 8. Let $\mathcal{G} = (A, R)$ be an argumentation framework, $a \in A$, and L be a labelling of $\mathcal{G}$.

- $L(a) = \mathbf{in}$ is legal if and only if for any $b \in A$, bRa implies $L(b) = \mathbf{out}$;
- $L(a) = \mathbf{out}$ is legal if and only if there exists $b \in A$ such that bRa and $L(b) = \mathbf{in}$;
- $L(a) = \mathbf{undec}$ is legal if and only if the above two cases are unsatisfied, i.e.
  - there exists $b \in A$ such that bRa and $L(b) \neq \mathbf{out}$;
  - for any $c \in A$, if cRa then $L(c) \neq \mathbf{in}$.

Based on Definition 8, various kinds of labellings can be defined as follows.

Definition 9. Let $\mathcal{G} = (A, R)$ be an argumentation framework, and L be a labelling of $\mathcal{G}$.

- L is an admissible labelling if and only if arguments in $\mathbf{in}(L)$ and $\mathbf{out}(L)$ are legally labeled by L;
- L is a complete labelling if and only if it is admissible, and arguments in $\mathbf{undec}(L)$ are legally labeled by L;
- L is the grounded labelling if and only if it is complete, and $\mathbf{in}(L)$ is minimal (with respect to set inclusion) among all complete labellings of $\mathcal{G}$;
- L is a preferred labelling if and only if it is admissible, and $\mathbf{in}(L)$ is maximal (with respect to set inclusion) among all admissible labellings of $\mathcal{G}$;
- L is a stable labelling if and only if it is complete, and $\mathbf{undec}(L) = \emptyset$;
- L is the ideal labelling if and only if it is the maximal admissible labelling that is smaller than or equal to each preferred labelling (with respect to set inclusion). Here, we say that a labelling L is smaller than or equal to another labelling L' if and only if $\mathbf{in}(L) \subseteq \mathbf{in}(L')$.

In this paper, we use ad, co, pr, gr, st and id to denote admissible, complete, preferred, grounded, stable and ideal respectively, and use σ to represent one of them, i.e. $\sigma \in \{ad, co, pr, gr, st, id\}$. The set of all σ-extensions(sets) of $\mathcal{G}$ is denoted as $\mathcal{E}_\sigma(\mathcal{G})$. The set of all σ-labellings on $\mathcal{G}$ is denoted as $\mathcal{L}_\sigma(\mathcal{G})$.

The relation between labellings and extensions is: for any σ-labelling of $\mathcal{G}$, there is a σ-extension(set) E such that $E = \mathbf{in}(L)$; for any σ-extension(set) E of $\mathcal{G}$, there is a σ-labelling such that $E = \mathbf{in}(L)$ ([1]). In the following part of this paper, we call $\mathbf{in}(L)$ a σ-extension(set) while L is a σ-labelling.

It is not the case for each σ in $\{ad, co, pr, gr, st, id\}$ that the σ-labellings are in one-to-one correspondence to the σ-extensions(sets) of an argumentation framework. ad-labellings are not uniquelly identified by their **in** labeled part, but it does hold for co-labellings ([12]). The following proposition indicates this unique identification for co-labellings.

Proposition 1. *Let $\mathcal{G} = (A, R)$ be an argumentation framework, and L_1, L_2 be co-labellings of $\mathcal{G}$. It holds that:*

- $\mathbf{in}(L_1) \subseteq \mathbf{in}(L_2)$ *if and only if* $\mathbf{out}(L_1) \subseteq \mathbf{out}(L_2)$*;*
- $\mathbf{in}(L_1) \subset \mathbf{in}(L_2)$ *if and only if* $\mathbf{out}(L_1) \subset \mathbf{out}(L_2)$*.*

2.3 Directionality and sub-frameworks

Directionality is a property of semantics with respect to the structures of argumentation frameworks. In this paper, we adopt the definition based on the labelling-based approach in [1].

Definition 10. Let $\mathcal{G} = (A, R)$ be an argumentation framework, and $B \subseteq A$. B is unattacked if and only if there is no argument $a \in A \setminus B$ such that aRB.

Definition 11. A semantics σ is directional if and only if for any argumentation framework $\mathcal{G}$ and for any set of arguments B which is unattacked, $\mathcal{L}_\sigma(\mathcal{G}) \downarrow_B = \mathcal{L}_\sigma(\mathcal{G} \downarrow_B)$, where $\mathcal{L}_\sigma(\mathcal{G}) \downarrow_B = \{L \downarrow_B \mid L \in \mathcal{L}_\sigma(\mathcal{G})\}$.

In [15], Liao et al. called $\mathcal{G} \downarrow_B$ with B unattacked unconditioned sub-framework of $\mathcal{G}$, otherwise conditioned sub-framework. Furthermore, they proposed the partially labeled sub-framework which is a combination of a conditioned sub-framework and its outside attackers. In this paper, we stick "partially labeled" to sub-framework $\mathcal{G} \downarrow_B$.

Definition 12. Let $\mathcal{G} = (A, R)$ be an argumentation framework, and $B \subseteq A$. A partially labeled sub-framework of $\mathcal{G}$ is denoted as $(\mathcal{G} \downarrow_B)^L$, where L is a labelling covers all attackers outside $\mathcal{G} \downarrow_B$.

In Definition 12, we do not restrict the attackers outside B to be nonempty, which is different from [15]. The legality of labellings for a partially labeled sub-framework needs to incorporate the labels of its external attackers if there are.

Definition 13. Let $\mathcal{G} = (A, R)$ be an argumentation framework, $B \subseteq A$, $a \in B$ and L^* be a labelling of $(\mathcal{G} \downarrow_B)^L$.

- $L^*(a) = \mathbf{in}$ is legal if and only if for any $b \in A \setminus B$, bRa implies $L(b) = \mathbf{out}$, and for any $c \in B$, bRa implies $L^*(c) = \mathbf{out}$;

- $L^*(a) = \mathbf{out}$ is legal if and only if there exists $b \in A \setminus B$, such that bRa and $L(b) = \mathbf{in}$ or there exists $b \in B$, such that bRa and $L^*(b) = \mathbf{in}$;
- $L^*(a) = \mathbf{undec}$ is legal if and only if the above two cases are unsatisfied.

The definitions of ad, co, gr, pr and id-labellings of a partially labeled sub-framework are defined as follows.

Definition 14. Let $\mathcal{G} = (A, R)$ be an argumentation framework, $B \subseteq A$ and L^* be a labelling of $(\mathcal{G} \downarrow_B)^L$.

- L^* is an ad-labelling of $(\mathcal{G} \downarrow_B)^L$ if and only if all arguments in $\mathbf{in}(L^*)$ and $\mathbf{out}(L^*)$ are legally labeled by L^*;
- L^* is a co-labelling of $(\mathcal{G} \downarrow_B)^L$ if and only if it is an ad-labelling, and all arguments in $\mathbf{undec}(L^*)$ are legally labeled by L^*;
- L^* is the gr-labelling of $(\mathcal{G} \downarrow_B)^L$, if and only if L^* is a co-labelling, and $\mathbf{in}(L^*)$ is minimal (with respect to set inclusion) among all co-labellings of $(\mathcal{G} \downarrow_B)^L$;
- L^* is a pr-labelling of $(\mathcal{G} \downarrow_B)^L$ if and only if L^* is an ad-labelling, and $\mathbf{in}(L^*)$ is maximal (with respect to set inclusion) among all ad-labellings of $(\mathcal{G} \downarrow_B)^L$;
- L^* is an st-labelling of $(\mathcal{G} \downarrow_B)^L$ if and only if it is a co-labeling, and $\mathbf{undec}(L)$ $= \emptyset$;
- L^* is the id-labelling of $(\mathcal{G} \downarrow_B)^L$ if and only if it is the maximal ad-labelling that is smaller than or equal to each pr-labelling of $(\mathcal{G} \downarrow_B)^L$ (with respect to set inclusion).

Example 4. $AF_3 \downarrow_{\{a,b\}}$ and $AF_3 \downarrow_{\{e,f\}}$ are two sub-frameworks of AF_3(see Figure 2). $AF_3 \downarrow_{\{a,b\}}$ is unconditioned, and given any labelling L, $\mathcal{L}_{co}((AF_3 \downarrow_{\{a,b\}})^L) = \{(\{a\}, \{b\}, \emptyset)\}$.

$AF_3 \downarrow_{\{e,f\}}$ is conditioned, and is attacked outside by c and d. Suppose $L_1 = (\{c\}, \{d\}, \emptyset)$, $L_2 = (\{d\}, \{c\}, \emptyset)$, and $L_3 = (\emptyset, \emptyset, \{c, d\})$, then we have:

$$\mathcal{L}_{co}((AF_3 \downarrow_{\{e,f\}})^{L_1}) = \{(\{f\}, \{e\}, \emptyset)\}$$
$$\mathcal{L}_{co}((AF_3 \downarrow_{\{e,f\}})^{L_2}) = \{(\{f\}, \{e\}, \emptyset)\}$$
$$\mathcal{L}_{co}((AF_3 \downarrow_{\{e,f\}})^{L_3}) = \{(\emptyset, \emptyset\, \{e, f\})\}$$

If a sub-framework is attacked outside by arguments that are all labeled **out**, then its semantics is imprevious. The following theorem shows this condition.

Theorem 2. $\sigma \in \{ad, co, gr, pr, st, id\}$. *Let* $\mathcal{G} = (A, R)$ *be an argumentation framework,* $B, C \subseteq A$*, and* $C = \{c \in A \mid c \notin B$ *and* $cRB\}$. *If there is a labelling L on C such that for any* $c \in C$, $L(c) = \mathbf{out}$, *then* $\mathcal{L}_\sigma((\mathcal{G} \downarrow_B)^L) = \mathcal{L}_\sigma(\mathcal{G} \downarrow_B)$.

It is easy to prove Theorem 2 by Definitions 13 and 14.

3 Characterizing Argumentation Frameworks

In this section, we will discuss how to characterize σ-argumentation frameworks with an extension. This kind of conditioned argumentation frameworks are defined following σ-subgraphs with respect to an extension proposed in [17]. A σ-subgraph with respect to an extension is a sub-framework that has a fixed set of arguments as a σ-extension. For example, AF_2, $AF_2 \downarrow_{\{a,b,c\}}$, $AF_2 \downarrow_{\{a,d,c\}}$ and $AF_2 \downarrow_{\{a,c\}}$ are all gr-subgraphs of AF_2 with respect to $\{a, c\}$(see Figure 1).

A σ-argumentation framework with an extension is defined as follows.

Definition 15. Let $\mathcal{G} = (A, R)$ be an argumentation framework. If E is a σ-extension of $\mathcal{G}$, then we call $\mathcal{G}$ a σ-argumentation framework with E. AF_E^σ is used to denote the set of all σ-argumentation frameworks with E.

AF_1 and AF_2 illustrated in Figure 1 are two gr-argumentation frameworks with $\{a, c\}$. AF_3 illustrated in Figure 2 is a co-argumentation framework with $\{a, c, f\}$.

E is the key point to the construction of $\mathcal{G}$ in AF_E^σ. Two factors related to E jointly make a contribution to this construction: arguments that attack E and arguments that are attacked by E. In the remaining part of this paper, the following sets related to E within any argumentation framework $\mathcal{G} = (A, R)$ will be frequently used, which are previously presented in [17].

$E_\mathcal{G}^- = \{x \in A \setminus E \mid xRE\}$, denoting the set of arguments in A that attacks E;
$E_\mathcal{G}^+ = \{x \in A \setminus E \mid ERx\}$, denoting the set of arguments in A that is attacked by E;
$E_\mathcal{G}^- \setminus E_\mathcal{G}^+$, denoting the set of arguments in A that attack E but is not attacked by E;
$I_\mathcal{G}^E = A \setminus (E \cup E_\mathcal{G}^- \cup E_\mathcal{G}^+)$, denoting the set of arguments in A which is unrelated to E (neither attacks nor is attacked by E).

E, $E_\mathcal{G}^- \setminus E_\mathcal{G}^+$, $E_\mathcal{G}^+$ and $I_\mathcal{G}^E$ make a partition of A, and determine whether $\mathcal{G}$ is a σ-argumentation framework with E.

The following parts of this section show the characterizations of argumentation frameworks with an extension under complete, grounded, preferred and stable semantics.

3.1 Complete semantics

Before discussing argumentation frameworks in AF_E^{co}, we first show how an ad-argumentation framework with E is.

Given an argumentation framework $\mathcal{G}$, if $\mathcal{G}$ is an ad-argumentation framework with E, then there exists an ad-labelling L of $\mathcal{G}$ such that **in**(L) is equal to E. All ad-sets are conflict-free, then any two arguments in E do not attack each other. According to Definition 9, arguments in **in**(L) and **out**(L) should be legally labeled. This implies that all arguments in $E^-_{\mathcal{G}}$ and $E^+_{\mathcal{G}}$ are labeled **out** by L. According to Definition 8, any argument labeled **out** is attacked by at least one argument labeled **in**. Then it can be concluded that E attacks $E^-_{\mathcal{G}}$, i.e. $E^-_{\mathcal{G}}$ is included in $E^+_{\mathcal{G}}$.

If $E^-_{\mathcal{G}}$ is included in $E^+_{\mathcal{G}}$ and E is conflict-free, then it can be concluded that $(E, E^+_{\mathcal{G}}, I^E_{\mathcal{G}})$ is an ad-labelling of $\mathcal{G}$. In this case, $\mathcal{G}$ is an ad-argumentation framework with E.

Theorem 3. *Let $\mathcal{G} = (A, R)$ be an argumentation framework, and $E \subseteq A$. $\mathcal{G} \in AF^{ad}_E$ if and only if*

- *$E \not R E$;*
- *$E^-_{\mathcal{G}} \subseteq E^+_{\mathcal{G}}$.*

Proof. ($\Rightarrow$) Suppose $\mathcal{G} \in AF^{ad}_E$, then there exists $L \in \mathcal{L}_{ad}(\mathcal{G})$ such that **in**$(L) = E$. Then **out**$(L) \subseteq E^+_{\mathcal{G}}$, since any arguments in **out**(L) should be legally labeled. From Definition 4, $E \not R E$. From Definitions 8 and 9, $E^-_{\mathcal{G}} \subseteq$ **out**(L). Then $E^-_{\mathcal{G}} \subseteq E^+_{\mathcal{G}}$.

($\Leftarrow$) Suppose $E \not R E$ and $E^-_{\mathcal{G}} \subseteq E^+_{\mathcal{G}}$, it is sufficient to prove that there exists $L \in \mathcal{L}_{ad}(\mathcal{G})$ such that **in**$(L) = E$. Let $L = (E, E^+_{\mathcal{G}}, A \setminus (E \cup E^+_{\mathcal{G}}))$ be a labelling of $\mathcal{G}$. For any $x \in E^+_{\mathcal{G}}$, there exists $y \in E$ such that yRx. Since $L(y) =$ **in**, then $L(x) =$ **out** is legal. Since $E^-_{\mathcal{G}} \subseteq E^+_{\mathcal{G}}$, then for any $x \in E^-_{\mathcal{G}}$, $L(x) =$ **out**. Since $E \not R E$, then for any $x \in E$, $L(x) =$ **in** is legal. From Definitions 8 and 9, $L \in \mathcal{L}_{ad}(\mathcal{G})$. Thus $\mathcal{G} \in AF^{ad}_E$. □

From Theorem 3, that $E^-_{\mathcal{G}}$ is included in $E^+_{\mathcal{G}}$ and that E is conflict-free are not only sufficient but also necessary conditions for $\mathcal{G}$ being an ad-argumentation framework with E.

Example 5. Let $E_2 = \{a, d\}$. According to Theorem 3, AF_4 illustrated in Figure 3 is an ad-argumentation framework with E_2.

AF_4

Figure 3: An ad-argumentation framework with E_2

Now let us turn to complete semantics. Given an argumentation framework $\mathcal{G}$, where $\mathcal{G} = (A, R)$, if $\mathcal{G}$ is a co-argumentation framework with E, then there exists a co-labelling L of $\mathcal{G}$ such that **in**(L) is equal to E. As a co-labelling is also an ad-labelling, $\mathcal{G}$ is first an ad-argumentation framework with E. Then from Theorem 3, we know that E is conflict-free and $E^-_{\mathcal{G}}$ is included in $E^+_{\mathcal{G}}$. From Definition 8, **out**(L) is equal to $E^+_{\mathcal{G}}$. From the construction of $I^E_{\mathcal{G}}$, we konw that $I^E_{\mathcal{G}}$ is equal to

$A \setminus (E \cup E^+_{\mathcal{G}})$. As **in**$(L)$ is equal to E and **out**(L) is equal to $E^+_{\mathcal{G}}$, $I^E_{\mathcal{G}}$ is just the set of **undec**(L). From Definition 9, arguments in **in**(L) and **out**(L) are legally labeled under complete semantics, and moreover, arguments in **undec**(L) are legally labeled. Then from Definition 8, all arguments in $I^E_{\mathcal{G}}$ are attacked by $I^E_{\mathcal{G}}$.

If E is conflict-free, $E^-_{\mathcal{G}}$ is included in $E^+_{\mathcal{G}}$, and all arguments in $I^E_{\mathcal{G}}$ is attacked by $I^E_{\mathcal{G}}$, then $(E, E^+_{\mathcal{G}}, I^E_{\mathcal{G}})$ makes a co-labelling of $\mathcal{G}$: arguments in E are legally labeled **in** since their attackers are in $E^+_{\mathcal{G}}$; arguments in $E^+_{\mathcal{G}}$ are legally labeled **out** because they are attacked by E; $I^E_{\mathcal{G}}$ is equal to $A \setminus (E \cup E^+_{\mathcal{G}})$ since $E^-_{\mathcal{G}}$ is included in $E^+_{\mathcal{G}}$; arguments in $I^E_{\mathcal{G}}$ are legally labeled **undec** because they are not attacked by E and not all attacked by arguments labeled **out**. Thus $\mathcal{G}$ is a co-argumentation framework with E.

On the basis of Theorem 3, a co-argumentation framework with E can be described as follows.

Theorem 4. *Let $\mathcal{G} = (A, R)$ be an argumentation framework, and $E \subseteq A$. $\mathcal{G} \in AF^{co}_E$ if and only if*

- $E \not R E$;
- $E^-_{\mathcal{G}} \subseteq E^+_{\mathcal{G}}$;
- *for any* $x \in I^E_{\mathcal{G}}$, $I^E_{\mathcal{G}} R x$.

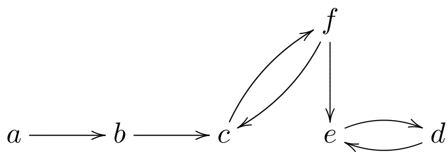
Proof. ($\Rightarrow$) Suppose $\mathcal{G} \in AF^{co}_E$, then there exists $L \in \mathcal{L}_{co}(\mathcal{G})$ such that **in**$(L) = E$. Since a co-labelling is also an ad-labelling, then $\mathcal{G} \in AF^{ad}_E$. Then $E \not R E$ and $E^-_{\mathcal{G}} \subseteq E^+_{\mathcal{G}}$. Then $I^E_{\mathcal{G}} = A \setminus (E \cup E^+_{\mathcal{G}})$. From Definition 8, **out**$(L) \subseteq E^+_{\mathcal{G}}$. If there is $x \in E^+_{\mathcal{G}}$ such that $L(x) \neq out$, then if $L(x) =$ **in**, then $x \in E$, a contradiction; if $L(x) =$ **undec**, then $E \not R x$, a contradiction. Then **out**$(L) = E^+_{\mathcal{G}}$, and then **undec**$(L) = I^E_{\mathcal{G}}$. Suppose there exists $x \in I^E_{\mathcal{G}}$ such that $I^E_{\mathcal{G}} \not R x$, then either x is unattacked or x is only attacked by $E \cup E^+_{\mathcal{G}}$. If x is unattacked, then $L(x) =$ **undec** is illegal, a contradiction. If x is attacked by $E \cup E^+_{\mathcal{G}}$, and since $L(x) =$ **undec**, then x only can be attacked by $E^+_{\mathcal{G}}$. Then $L(x) =$ **undec** is illegal, a contradiction. Thus, for any $x \in I^E_{\mathcal{G}}$, $I^E_{\mathcal{G}} R x$.

($\Leftarrow$) Suppose $E \not R E$, $E^-_{\mathcal{G}} \subseteq E^+_{\mathcal{G}}$ and for any $x \in I^E_{\mathcal{G}}$, $I^E_{\mathcal{G}} R x$, it is sufficient to prove that there exists $L \in \mathcal{L}_{co}(\mathcal{G})$ such that **in**$(L) = E$. Let $L = (E, E^+_{\mathcal{G}}, A \setminus (E \cup E^+_{\mathcal{G}}))$ be a labelling of $\mathcal{G}$. From $E \not R E$ and $E^-_{\mathcal{G}} \subseteq E^+_{\mathcal{G}}$, we know that $\mathcal{G} \in AF^{ad}_E$, and then arguments in E and $E^+_{\mathcal{G}}$ are legally labeled. Since $E^-_{\mathcal{G}} \subseteq E^+_{\mathcal{G}}$, then $I^E_{\mathcal{G}} = A \setminus (E \cup E^+_{\mathcal{G}})$. Since for any $x \in I^E_{\mathcal{G}}$, $I^E_{\mathcal{G}} R x$, then for any $x \in A \setminus (E \cup E^+_{\mathcal{G}})$, x is not attacked by **in**(L), and is not only attacked by **out**(L). Then $L(x) =$ **undec** is legal. Then for any $x \in A \setminus (E \cup E^+_{\mathcal{G}})$, $L \in \mathcal{L}_{co}(\mathcal{G})$. Thus $\mathcal{G} \in AF^{co}_E$. □

Comparing with Theorem 3, Theorem 4 adds a new condition on $I^E_{\mathcal{G}}$ which makes sure that all arguments unrelated to E are legally labeled **undec**. Theorem 4 provides sufficient and necessary conditions for $\mathcal{G}$ being a co-argumentation framework with E.

Example 6. $E_2 = \{a, d\}$, and AF_4 in figure 3 is an ad-argumentation framework with E_2. $I^{E_2}_{AF_4} = \{c, f\}$. According to Theorem 4, all arguments in $\{c, f\}$ should be attacked by $\{c, f\}$. Thus AF_4 is not in AF^{co}_E. Both AF^1_5 and AF^2_5 illustrated in Figure 4 are co-argumentation frameworks with E_2.

AF^1_5



AF^2_5

Figure 4: co-argumentation frameworks with E_2

3.2 Grounded semantics

If $\mathcal{G}$ is a gr-argumentation framework with E, then there is a gr-labelling of $\mathcal{G}$, say L, such that **in**(L) is equal to E. As the gr-labelling is also complete, $\mathcal{G}$ first is a co-argumentation framework with E. Then we know that E is conflict-free, $E^-_{\mathcal{G}}$ is included in $E^+_{\mathcal{G}}$, and $I^E_{\mathcal{G}}$ is self-attacked. As the grounded extension is the minimal complete extension, $\mathcal{G}$ should have no other co-extension that is properly included in E. This implies that the sub-framework $\mathcal{G}\downarrow_{E\cup E^+_{\mathcal{G}}}$ should not have arguments that can be labeled **undec**, i.e. $(E, E^+_{\mathcal{G}}, \emptyset)$ is the gr-labeling of $\mathcal{G}\downarrow_{E\cup E^+_{\mathcal{G}}}$.

Suppose $\mathcal{G}$ is a co-argumentation framework with E, and $(E, E^+_{\mathcal{G}}, \emptyset)$ is the gr-labelling of $\mathcal{G}\downarrow_{E\cup E^+_{\mathcal{G}}}$, then $(E, E^+_{\mathcal{G}}, I^E_{\mathcal{G}})$ is a co-labelling of $\mathcal{G}$, and there is no smaller co-extension of $\mathcal{G}$ than E. Then $(E, E^+_{\mathcal{G}}, I^E_{\mathcal{G}})$ becomes the gr-labelling of $\mathcal{G}$, and $\mathcal{G}$ is a gr-argumentation framework with E.

In [19], Modgil and Caminada provided an algorithm to compute the gr-labelling of an argumentation framework. The algorithm started by assigning **in** to all arguments that are unattacked, and then iteratively assign **out** to any argument that is attacked by an argument which has been assigned **in**, and **in** to those arguments whose attackers are all assigned **out**. The iteration continues until no more new arguments can be assigned **in** or **out**, then all the arguments left are assigned **undec**. In this process of assignment, for any circle, if it is unattacked or attacked only by arguments assigned **out**, then all arguments in it can only be decided to be **undec**. But if it is attacked by arguments assigned **in**, then some arguments in this circle can be decided to be **in** or **out**.

Example 7. AF_6^1 and AF_6^2 are argumentation frameworks with circles (see Figure 5). The assigning process of grounded semantics for AF_6^1 is: $a(\textbf{in}) \rightarrow b(\textbf{out}) \rightarrow c(\textbf{undec})$. The assigning process of grounded semantics for AF_6^2 is: $a(\textbf{in}) \rightarrow b(\textbf{out}) \rightarrow c(\textbf{in})$.

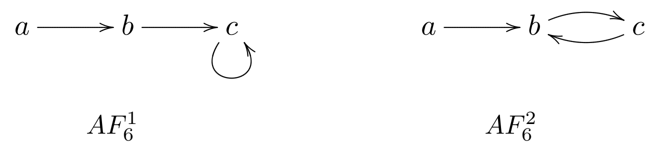


Figure 5: Argumentation frameworks with circles

The algorithm of computing gr-labelling provides a simple way to check whether $(E, E_{\mathcal{G}}^+, \emptyset)$ is the gr-labelling of $\mathcal{G}\downarrow_{E\cup E_{\mathcal{G}}^+}$, that is: whether all circles in $\mathcal{G}\downarrow_{E\cup E_{\mathcal{G}}^+}$ are attacked outside by E. If all circles in $\mathcal{G}\downarrow_{E\cup E_{\mathcal{G}}^+}$ are attacked outside by E, then $(E, E_{\mathcal{G}}^+, \emptyset)$ is the gr-labelling of $\mathcal{G}\downarrow_{E\cup E_{\mathcal{G}}^+}$. If not, then arguments of some circles in $\mathcal{G}\downarrow_{E\cup E_{\mathcal{G}}^+}$ are labeled **undec**, which makes $(E, E_{\mathcal{G}}^+, \emptyset)$ not the minimal co-labelling of $\mathcal{G}\downarrow_{E\cup E_{\mathcal{G}}^+}$.

Consider argumentation frameworks in Figure 5, in the case of $E_1 = \{a, c\}$, both $E_1 \cup {E_1}^+_{AF_6^1}$ and $E_1 \cup {E_1}^+_{AF_6^2}$ are equal to $\{a, b, c\}$. $AF_6^2\downarrow_{\{b,c\}}$ is a circle and it is attacked outside by a in E_1, and AF_6^2 has $(\{a, c\}, \{b\}, \emptyset)$ as its gr-labelling. $AF_6^1\downarrow_{\{c\}}$ is a circle. $AF_6^1\downarrow_{\{c\}}$ is attacked by c, but c is not outside $AF_6^1\downarrow_{\{c\}}$, i.e. c is not in $E \setminus \{c\}$. AF_6^1 has $(\{a\}, \{b\}, \{c\})$ as its gr-labelling.

Based on Theorem 4 and the algorithm of computing gr-labellings, a gr-argumentation framework with E can be described as follows.

Theorem 5. *Let $\mathcal{G} = (A, R)$ be an argumentation framework, and $E \subseteq A$. $\mathcal{G} \in AF_E^{gr}$ if and only if*

- $E \not R E$;
- $E_{\mathcal{G}}^- \subseteq E_{\mathcal{G}}^+$;
- *for any* $x \in I_{\mathcal{G}}^E$, $I_{\mathcal{G}}^E Rx$.
- *for any* $C \in SCIR_{\mathcal{G}\downarrow_{E\cup E_{\mathcal{G}}^+}}$, $(E \setminus C)RC$.

Proof. ($\Rightarrow$) Suppose $\mathcal{G} \in AF_E^{gr}$, then there exists $L \in \mathcal{L}_{gr}(\mathcal{G})$ such that $\textbf{in}(L) = E$. Since a gr-labelling is also a co-labelling, then $\mathcal{G} \in AF_E^{co}$. Then $E \not R E$, $E_{\mathcal{G}}^- \subseteq E_{\mathcal{G}}^+$ and for any $x \in I_{\mathcal{G}}^E$, $I_{\mathcal{G}}^E Rx$. From the proof of Theorem 4, we know that $\textbf{out}(L) = E_{\mathcal{G}}^+$ and $\textbf{undec}(L) = I_{\mathcal{G}}^E$. $L\downarrow_{E\cup E_{\mathcal{G}}^+}$ is the gr-labelling of $\mathcal{G}\downarrow_{E\cup E_{\mathcal{G}}^+}$, since if not, L will not be the gr-labelling of $\mathcal{G}$. Suppose there exists $V \in SCIR_{\mathcal{G}\downarrow_{E\cup E_{\mathcal{G}}^+}}$ such that

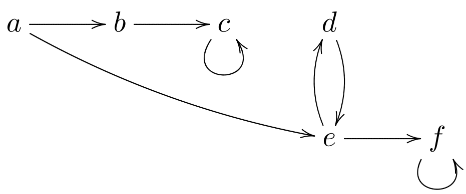
$(E \setminus V)\not{R}V$, then either V is unattacked, or V is attacked by $E^+_{\mathcal{G}} \setminus V$. In each case, there is another co-labelling, say L', of $\mathcal{G} \downarrow_{E\cup E^+_{\mathcal{G}}}$ such that $\mathbf{in}(L') \subset \mathbf{in}(L \downarrow_{E\cup E^+_{\mathcal{G}}})$, contradicting that $L \downarrow_{E\cup E^+_{\mathcal{G}}}$ is the gr-labelling of $\mathcal{G} \downarrow_{E\cup E^+_{\mathcal{G}}}$. Thus, for any $C \in SCIR_{\mathcal{G}\downarrow_{E\cup E^+_{\mathcal{G}}}}$, $(E \setminus C)RC$.

($\Leftarrow$) Suppose the four conditions are satisfied, it is sufficient to prove that there exists $L \in \mathcal{L}_{gr}(\mathcal{G})$ such that $\mathbf{in}(L) = E$. Let $L = (E, E^+_{\mathcal{G}}, A \setminus (E \cup E^+_{\mathcal{G}}))$ be a labelling of $\mathcal{G}$. $E\not{R}E$, $E^-_{\mathcal{G}} \subseteq E^+_{\mathcal{G}}$ and that for any $x \in I^E_{\mathcal{G}}$, $I^E_{\mathcal{G}}Rx$ implies that $\mathcal{G} \in AF^{co}_E$. Then L is a co-labelling of $\mathcal{G}$. From Definitions 8 and 9, $L \downarrow_{E\cup E^+_{\mathcal{G}}}$ is a co-labelling of $\mathcal{G} \downarrow_{E\cup E^+_{\mathcal{G}}}$. Suppose $L' \in \mathcal{L}_{gr}(\mathcal{G})$ and $L' \neq L$, then $\mathbf{in}(L') \subset E$. Then from Proposition 1, $\mathbf{out}(L') \subset \mathbf{out}(L)$. Then there exists $U \subseteq E \cup (E^+_{\mathcal{G}})$ such that for any $x \in U$, $L'(x) = \mathbf{undec}$. The algorithm of computing gr-labelling in [19] indicates that at least one circle is included in U. Then there is $V \in SCIR_{\mathcal{G}\downarrow_{E\cup E^+_{\mathcal{G}}}}$ such that for any $y \in V^-_{\mathcal{G}\downarrow_{E\cup E^+_{\mathcal{G}}}}$, $L'(y) = \mathbf{out}$. Then $(E \setminus V)\not{R}V$, contradicting that for any $C \in SCIR_{\mathcal{G}\downarrow_{E\cup E^+_{\mathcal{G}}}}$, $(E \setminus C)RC$. Then L is the gr-labelling of $\mathcal{G}$. Thus $\mathcal{G}$ is a gr-argumentation framework with E. □

Theorem 5 adds a new condition on circles in $\mathcal{G} \downarrow_{E\cup E^+_{\mathcal{G}}}$ to Theorem 4 to make $\{E, E^+_{\mathcal{G}}, I^E_{\mathcal{G}}\}$ from a co-labelling to be the gr-labelling of $\mathcal{G}$. It provides sufficient and necessary conditions for $\mathcal{G}$ being a gr-argumentation framework with E.

Example 8. $E_2 = \{a, d\}$ and AF^1_5 and AF^2_5 illustrated in Figure 4 are co-argumentation frameworks with E_2. $AF^1_5 \downarrow_{\{d,e\}}$ is a circle and $\{d, e\}^-_{AF^1_5}$ is empty. According to Theorem 5, AF^1_5 is not a gr-argumentation framework with E_2. $AF^2_5 \downarrow_{\{d,e\}}$ is a circle and $\{d, e\}^-_{AF^2_5} = \{f\}$. f is obviously not in $E_2 \setminus \{d, e\}$. According to Theorem 5, AF^2_5 is not a gr-argumentation framework with E_2.

The argumentation framework AF_7 presented in Figure 6 is in accord with Theorem 5 in which $AF_7 \downarrow_{\{d,e\}}$ is attacked by a. AF_7 is a gr-argumentation framework with E_2.



AF_7

Figure 6: A gr-argumentation framework with E_2

3.3 Preferred semantics

Given an argumentation framework $\mathcal{G}$, if $\mathcal{G}$ is a pr-argumentation framework with E, then there exists a pr-labelling of $\mathcal{G}$, say L, such that **in**(L) is equal to E. As a pr-labelling is also admissible, $\mathcal{G}$ first is an ad-argumentation framework with E. Then we know that $E^-_\mathcal{G}$ is included in $E^+_\mathcal{G}$, and then **out**(L) is equal to $E^+_\mathcal{G}$. Furthermore, $I^E_\mathcal{G}$ is equal to $A_\mathcal{G} \setminus (E \cup E^+_\mathcal{G})$. As any pr-extension of $\mathcal{G}$ is a maximal ad-set, then there are no more arguments in $I^E_\mathcal{G}$ that can be labeled **in** by all ad-labellings of $\mathcal{G}$. Confining it to the sub-framework $\mathcal{G} \downarrow_{I^E_\mathcal{G}}$, there is only one ad-labelling of $\mathcal{G} \downarrow_{I^E_\mathcal{G}}$ which is $(\emptyset, \emptyset, I^E_\mathcal{G})$.

If $\mathcal{G}$ is an ad-argumentation framework with E, then $(E, E^+_\mathcal{G}, I^E_\mathcal{G})$ is an ad-labelling of $\mathcal{G}$. With another premise that $(\emptyset, \emptyset, I^E_\mathcal{G})$ is the only ad-labelling of $\mathcal{G} \downarrow_{I^E_\mathcal{G}}$, E makes a maximal ad-set of $\mathcal{G}$. Then that $(E, E^+_\mathcal{G}, I^E_\mathcal{G})$ is also a pr-labelling of $\mathcal{G}$, and $\mathcal{G}$ is a pr-argumentation framework with E.

The following theorem shows how to characterize argumentation frameworks in AF^{pr}_E.

Theorem 6. *Let* $\mathcal{G} = (A, R)$ *be an argumentation framework, and* $E \subseteq A$. $\mathcal{G} \in AF^{pr}_E$ *if and only if*

- $E \not{R} E$.
- $E^-_\mathcal{G} \subseteq E^+_\mathcal{G}$.
- $\mathcal{L}_{ad}(\mathcal{G} \downarrow_{I^E_\mathcal{G}}) = \{(\emptyset, \emptyset, I^E_\mathcal{G})\}$.

Proof. ($\Rightarrow$) Suppose $\mathcal{G} \in AF^{pr}_E$, then there exists $L \in \mathcal{L}_{pr}(\mathcal{G})$ such that **in**$(L) = E$. Since a pr-labelling is an ad-labelling, then $\mathcal{G}$ is an ad-argumentation framework with E. Then $E^-_\mathcal{G} \subseteq E^+_\mathcal{G}$, $E \not{R} E$ and $I^E_\mathcal{G} = A \setminus (E^+_\mathcal{G} \cup E)$. Then for any $y \in (I^E_\mathcal{G})^-_\mathcal{G}$, $y \in E^+_\mathcal{G}$. Suppose there exists $L_1 \in \mathcal{L}_{ad}(\mathcal{G} \downarrow_{I^E_\mathcal{G}})$ such that **in**$(L_1) \neq \emptyset$, then from Definitions 8 and 9, $L' = L \downarrow_{E \cup E^+_\mathcal{G}} \cup L_1$ is an ad-labelling of $\mathcal{G}$. Then **in**$(L') \supset$ **in**(L), contradicting that L is a pr-labelling of $\mathcal{G}$. Thus, $\mathcal{L}_{ad}(\mathcal{G} \downarrow_{I^E_\mathcal{G}}) = \{(\emptyset, \emptyset, I^E_\mathcal{G})\}$.

($\Leftarrow$) Suppose $E \not{R} E$, $E^-_\mathcal{G} \subseteq E^+_\mathcal{G}$, and $\mathcal{L}_{ad}(\mathcal{G} \downarrow_{I^E_\mathcal{G}}) = \{(\emptyset, \emptyset, I^E_\mathcal{G})\}$, it is sufficient to prove that there exists $L \in \mathcal{L}_{pr}(\mathcal{G})$ such that **in**$(L) = E$. Let $L = (E, E^+_\mathcal{G}, A \setminus (E \cup E^+_\mathcal{G}))$ be a labelling of $\mathcal{G}$. Since $E \not{R} E$ and $E^-_\mathcal{G} \subseteq E^+_\mathcal{G}$, then $\mathcal{G}$ is an ad-argumentation framework with E. Then L is an ad-labelling of $\mathcal{G}$. Then $I^E_\mathcal{G} = A \setminus (E \cup E^+_\mathcal{G})$, and $I^E_\mathcal{G}$ is only be attacked outside by $E^+_\mathcal{G}$. From Proposition 2, $\mathcal{L}_{ad}(\mathcal{G} \downarrow_{I^E_\mathcal{G}}) = \mathcal{L}_{ad}((\mathcal{G} \downarrow_{I^E_\mathcal{G}})^L)$. Since $\mathcal{L}_{ad}(\mathcal{G} \downarrow_{I^E_\mathcal{G}}) = \{(\emptyset, \emptyset, I^E_\mathcal{G})\}$, then there is no more argument in $I^E_\mathcal{G}$ that can be labeled **in** or **out**. Then L is a pr-labelling of $\mathcal{G}$. Thus $\mathcal{G}$ is a pr-argumentation framework with E. □

Theorem 6 provides sufficient and necessary conditions for $\mathcal{G}$ being a pr-argumentation framework with E. Unlike that of complete and grounded semantics, we can not judge whether an argumentation framework is in AF_E^{pr} only by its structure. The only way we know now to check whether $\mathcal{L}_{ad}(\mathcal{G}\downarrow_{I_\mathcal{G}^E})$ is $\{(\emptyset,\emptyset,I_\mathcal{G}^E)\}$ is to compute $\mathcal{L}_{ad}(\mathcal{G}\downarrow_{I_\mathcal{G}^E})$.

Example 9. $E_2=\{a,d\}$. AF_4 shows in Figure 3 is an ad-argumentation framework but not a pr-argumentation framework with E_2, since c and f are not labeled **undec** in $AF_4\downarrow_{\{c,f\}}$. AF_5^1 illustrated in Figure 4 is a pr-argumentation framework with E_2, and we can see that its sub-framework $AF_5^1\downarrow_{\{c,f\}}$ has no arguments accepted. Either c or f is accepted in $AF_5^2\downarrow_{\{c,f\}}$, thus AF_5^2 illustrated in Figure 4 is not a pr-argumentation framework with E_2.

3.4 Stable semantics

Given an argumentation framework $\mathcal{G}$, if $\mathcal{G}$ is an st-argumentation framework with E, then there exists an st-labelling L of $\mathcal{G}$ such that **in**(L) is equal to E. As an st-labelling is also an ad-labelling, $\mathcal{G}$ first is an ad-argumentation framework with E. Then from Theorem 3, $E\not{R}E$ and $E_\mathcal{G}^-$ is included in $E_\mathcal{G}^+$. Then **out**(L) is equal to $E_\mathcal{G}^+$, and **undec**(L) is equal to $I_\mathcal{G}^E$. The speciality of an st-labelling is that no arguments are labeled **undec**. This makes **undec**(L) be empty, i.e. $I_\mathcal{G}^E$ is an empty set.

If $\mathcal{G}$ is an ad-argumentation framework with E and $I_\mathcal{G}^E$ is empty, it is easy to see that $(E,E_\mathcal{G}^+,I_\mathcal{G}^E)$ is not only an ad-labelling but also an st-labelling of $\mathcal{G}$.

Based on Theorem 3, argumentation frameworks in AF_E^{st} are constructed as follows.

Theorem 7. *Let $\mathcal{G}=(A,R)$ be an argumentation framework, and $E\subseteq A$. $\mathcal{G}\in AF_E^{st}$ if and only if*

- $E\not{R}E$;
- $E_\mathcal{G}^-\subseteq E_\mathcal{G}^+$;
- $I_\mathcal{G}^E=\emptyset$.

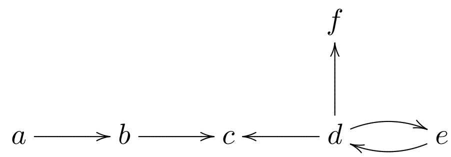
Proof. ($\Rightarrow$) Suppose $\mathcal{G}\in AF_E^{st}$, then there exists $L\in\mathcal{L}_{st}(\mathcal{G})$ such that **in**$(L)=E$. Since an st-labelling is also an ad-lebelling, then $\mathcal{G}$ is an ad-argumentation framework with E, and **undec**$(L)=\emptyset$. Then $E\not{R}E$ and $E_\mathcal{G}^-\subseteq E_\mathcal{G}^+$. Then **out**$(L)=E_\mathcal{G}^+$ and **undec**$(L)=I_\mathcal{G}^E$. Since **undec**$(L)=\emptyset$, then $I_\mathcal{G}^E=\emptyset$.

($\Leftarrow$) Suppose $E\not{R}E$, $E_\mathcal{G}^-\subseteq E_\mathcal{G}^+$, and $I_\mathcal{G}^E=\emptyset$, it is sufficient to prove that there exists $L\in\mathcal{L}_{st}(\mathcal{G})$ such that **in**$(L)=E$. Let $L=(E,E_\mathcal{G}^+,A\backslash(E\cup E_\mathcal{G}^+))$ be a labelling of $\mathcal{G}$. Since $E\not{R}E$ and $E_\mathcal{G}^-\subseteq E_\mathcal{G}^+$, then $\mathcal{G}$ is an ad-argumentation framework with E, and $A\setminus(E\cup E_\mathcal{G}^+)=I_\mathcal{G}^E$. Then L is an ad-labelling of $\mathcal{G}$. Since $I_\mathcal{G}^E=\emptyset$, then $A\setminus(E\cup E_\mathcal{G}^+)=\emptyset$. Then L is an st-labelling of $\mathcal{G}$. Thus $\mathcal{G}$ is an st-argumentation framework with E. □

Theorem 7 provides sufficient and necessary conditions for $\mathcal{G}$ being an st-argumentation framework with E.

Example 10. $E_2 = \{a, d\}$. AF_4 in Figure 3 is an ad-argumentation framework with E_2. One of the ad-labellings of AF_4 is $(\{a, d\}, \{b, e\}, \{c, f\})$. Since $I^E_{AF_4} = \{c, f\}$, then from Theorem 7, AF_4 is not an st-argumentation framework with E_2. AF^1_8 and AF^2_8 illustrated in Figure 7 are qualified argumentation frameworks in $AF^{st}_{E_2}$.

AF^1_8



AF^2_8

Figure 7: st-argumentation frameworks with E_2

We call all above theorems for the characterizations of argumentation frameworks in AF^{σ}_E chracterizing theorems, where $\sigma \in \{ad, co, gr, pr, st\}$.

3.5 Properties of σ-argumentation frameworks with E

There is a series of inclusion relations between admissible sets, complete, grounded, preferred, stable and ideal extensions of an argumentation framework. They actually indicate a kind of ordering on these semantics if we treat “admissible” also as a kind of semantics.

Definition 16. Let σ and τ be two kinds of semantics, and $\sqsubseteq$ be a relation between them. $\sigma \sqsubseteq \tau$ if and only if for any argumentation framework $\mathcal{G}$, for any $E \in \mathcal{E}_\sigma(\mathcal{G})$, $E \in \mathcal{E}_\tau(\mathcal{G})$.

The fact that every stable extension is also a preferred extension was first stated in [18]. All other relations between admissible sets, complete, grounded and preferred extensions have originally been stated in [11]. Meanwhile, as proved in [8], an ideal extension is also a complete extension. All these indicates that $\sqsubseteq$ is an odering on admissible, complete, grounded, preferred, stable and ideal semantics:

$st \sqsubseteq pr \sqsubseteq co \sqsubseteq ad$;

$gr \sqsubseteq co \sqsubseteq ad$;
$id \sqsubseteq co \sqsubseteq ad$.

Given a set of arguments E, this odering implies inclusion relations between corresponding sets of argumentation frameworks with E.

Theorem 8. *Let E be a set of arguments. Then we know that:*

- $AF_E^{st} \subseteq AF_E^{pr} \subseteq AF_E^{co} \subseteq AF_E^{ad}$
- $AF_E^{gr} \subseteq AF_E^{co} \subseteq AF_E^{ad}$
- $AF_E^{id} \subseteq AF_E^{co} \subseteq AF_E^{ad}$

The definitions of AF_E^{σ} and $\sqsubseteq$ makes a clear clue to prove Theorem 8.

At last, we extend the chatacterization of σ-argumentation frameworks with a single extension to a set of extensions.

Theorem 9. *Let $\mathcal{G} = (A, R)$ be an argumentation framework, and $\mathcal{B} = \{B \mid B \subseteq A\}$. $\mathcal{B} \subseteq \mathcal{E}_{\sigma}(\mathcal{G})$ if and only if $\mathcal{G} \in \bigcap\{AF_E^{\sigma} \mid E \in \mathcal{B}\}$.*

Theorem 9 can be proved directly from the definition of AF_E^{σ}.

4 Applications

In this section, we will discuss two applications of our work in Section 3 on the dynamics of argumentation frameworks. One application is updating an argumentation framework to enforce an extension. The other one is monotony.

4.1 Updating an argumentation framework to enfroce an extension

Updating an argumentation framework is to change the set of arguments and the attack relation of this tuple. In [16], Liao et al. treated it as an operation between an arguementation framework and a set of arguments and attacks. In this paper, we adopt the perspective that updating an argumentation framework is adding arguments or attacks to it, or deleting arguments or attacks from it, and after revising, it still becomes an argumentation framework.

Definition 17. Let $\mathcal{G} = (A, R)$ be an argumentation framework. The update of $\mathcal{G}$ is an argumentation framework $\mathcal{G}' = (A', R')$, and $\mathcal{G}'$ satisfies

- $A' = (A \setminus B) \cup C$ where $B \subseteq A$ and $C \cap (A \setminus B) = \emptyset$;
- $R' = (R \setminus R^1) \cup R^2$ where R^1 and R^2 are two binary relations, $(A \times B) \cup (B \times B) \cup (B \times A) \subseteq R^1$ and $R^2 \subseteq A' \times A'$.

Updating an argumentation framework to enforce an extension, say E, means updating an argumentation framework in such a way that E becomes one of its extensions. In [4], Baunman et al. have found the sufficient conditions for expanding an argumentation framework, i.e. adding arguments or attaks to it, to enforce an extension. In this paper, each characterizing theorem in Section 3 provides a way to update an argumentation framework, either expanding or restricting, to enforce a σ-extension where $\sigma \in \{co, gr, pr, st\}$.

Theorems 4, 5 and 7 show direct ways to update an argumentation framework to enforce a complete, grounded and stable extension, respectively.

Example 11. Consider AF_1 in Figure 1. Let $E_3 = \{a\}$. E_3 is an ad-set but not a co, gr or st-extension of AF_1. AF_1^1 and AF_1^2 are the updated argumentation frameworks of AF_1 that have E_3 as a co-extension (see Figure 8). AF_1^1 is constructed by adding (b, a) and (c, c) to AF_1. $I^{E_3}_{AF_1^1} = \{c\}$, and it is self-attacked. AF_1^2 is constructed by deleting c, (b, c) and (c, b) from AF_1, and add (b, a) to it. $I^{E_3}_{AF_1^2} = \emptyset$, and it absolutely satisfies Theorem 4. From Theorem 7, E_3 is also an st-extension of AF_1^2.

The circle $AF_1^1 \downarrow_{\{a,b\}}$ is not attacked by $E_3 \setminus \{a, b\}$, then from Theorem 5, E_3 is not the gr-extension of AF_1^1. AF_1^3 has E_3 as the gr-extension which is constructed by delete (b, a) from AF_1^1 (see Figure 8). After updating, there is no circle in $\mathcal{G} \downarrow_{E_3 \cup {E_3}^+_{AF_1^3}}$, and it absolutly satisfies Theorem 5.

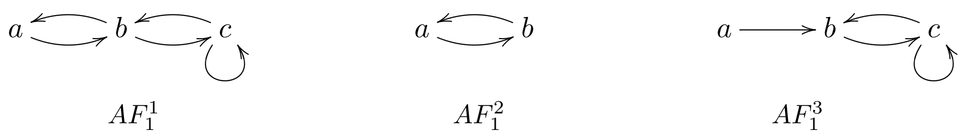
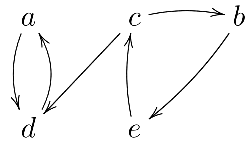


Figure 8: The update of AF_1 to enforce E_3

Theorem 6 shows an indirect way to update an argumentation framework to enforce a preferred extension.

Example 12. $E_3 = \{a\}$, and E_3 is not a pr-extension but an ad-set of AF_2 (see Figure 1). AF_2^1 is an updated argumentation framework of AF_2 to enforce E_3 as a pr-extension (see Figure 9). AF_2^1 is formed by adding e, (e, c), (b, e) to AF_2, where $I^{E_3}_{AF_2^1} = \{b, c, e\}$ and there is no arguments in it can be labeled **in**.

Examples 11 and 12 show how to update an argumentation framework to enforce an extension. The general rules of these updates under complete, grounded, stable and preferred semantics are displayed in [21], and each rule corresponds to a characterizing theorem of the same semantics in Section 3. The ways to update argumentation frameworks in Examples 11 and 12 conform to the rules in [21], while the resulted frameworks satisfy the characterizing theorems.



AF_2^1

Figure 9: The update of AF_2 to enforce E_3

4.2 Monotony

Monotony is an important conception in mathematics and logic. In abstract argumentation, monotony is a kind of property of the update from an argumentation framework, say $\mathcal{G}$, to the other one, say $\mathcal{G}'$, that represents the monotonic change of accepted arguments. In [10], Cayrol et al. proposed monotony, credulous monotony and skeptical monotony. Monotony indicates each extension of $\mathcal{G}$ is included in at least one extension of $\mathcal{G}'$. Credulous monotony indicates the union of the extensions of $\mathcal{G}$ is included in the union of the extensions of $\mathcal{G}'$. Skeptical monotony indicates the intersection of extensions of $\mathcal{G}$ is included in the intersection of extensions of $\mathcal{G}'$. In [5], monotony is classified as expansive monotony and restrictive monotony. The update from $\mathcal{G}$ to $\mathcal{G}'$ is expansive monotony if every argument accepted in $\mathcal{G}$ is still accepted in $\mathcal{G}'$, i.e. no accepted argument is lost or there is an expansion of acceptability. The update from $\mathcal{G}$ to $\mathcal{G}'$ is restrictive monotony if every argument accepted in $\mathcal{G}'$ was already accepted in $\mathcal{G}$, i.e. no new acceptable arguments appear or there is a restriction of acceptability. In [4], monotony means that arguments accepted in the original argumentation framework survive, and the number of extensions can not decrease after updating.

In this paper we discuss some relations between the σ-argumentation frameworks with an extension and the monotonies proposed in [5]. The definition of monotonies is defined as follows.

Definition 18. Let $\mathcal{G} = (A, R)$ and $\mathcal{G}' = (A', R')$ be argumentation frameworks.

- The update from $\mathcal{G}$ to $\mathcal{G}'$ is expansive σ-monotony if and only if for any σ-extension E of $\mathcal{G}$, there is a σ-extension E' of $\mathcal{G}'$ such that $E \subseteq E'$.
- The update from $\mathcal{G}$ to $\mathcal{G}'$ is restrictive σ-monotony if and only if for any σ-extension E of $\mathcal{G}$, there is a σ-extension E' of $\mathcal{G}'$ such that $E \supseteq E'$.

The σ-argumentation framework with an extension can be used to show the conditions for monotony.

From Definition 6, we know that $\subseteq$ is an ordering on the set of co-extensions of an argumentation framework. If all the maximal co-extensions, i.e. all pr-extensions of the original argumentation framework $\mathcal{G}$ survive in the updated argumentation framework $\mathcal{G}'$, then all co-extensions survive. Then the expansive monotony under complete and preferred semantics are satisfied. Since both the gr-extension and the id-extension are unique in a framework, then the expansive monotony under grounded and ideal semantics are satisfied if the corresponding gr and id-extensions survives.

The following theorem uses the set of σ-argumentation frameworks with an extension to show some sufficient conditions for expansive monotony.

Theorem 10. *Let $\mathcal{G} = (A, R)$ and $\mathcal{G}' = (A', R')$ be argumentation frameworks.*

- *If $\mathcal{G}' \in \bigcap\{AF_E^{ad} | E \in \mathcal{E}_{pr}(\mathcal{G})\}$, then the update from $\mathcal{G}$ to $\mathcal{G}'$ is expansive co and pr-monotony.*
- *If $\mathcal{G}' \in AF_E^{gr}$ where $E \in \mathcal{E}_{id}(\mathcal{G})$, then the update from $\mathcal{G}$ to $\mathcal{G}'$ is expansive gr and id-monotony.*

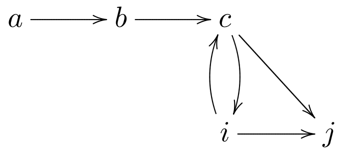
Proof.

- Suppose $\mathcal{G}' \in \bigcap\{AF_E^{ad} | E \in \mathcal{E}_{pr}(\mathcal{G})\}$, then any pr-extension E of $\mathcal{G}$ is an ad-set of $\mathcal{G}'$. For any co-extension E^1 of $\mathcal{G}$, there is a pr-extension E^2 of $\mathcal{G}$ such that $E^1 \subseteq E^2$. Since E^2 is an ad-set of $\mathcal{G}'$, then there must be a co-extension E^3 of $\mathcal{G}'$ such that $E^2 \subseteq E^3$. Thus the update from $\mathcal{G}$ to $\mathcal{G}'$ is expansive co-monotony.
  The pr-extension is also a co-extension, then the update from $\mathcal{G}$ to $\mathcal{G}'$ is also expansive pr-monotony.
- Suppose $\mathcal{G}' \in AF_E^{gr}$ and $E \in \mathcal{E}_{id}(\mathcal{G})$, then the id-extension E of $\mathcal{G}$ is the gr-extension of $\mathcal{G}'$. Since the gr-extension is included in the id-extension of an argumentation framework, then E includes the gr-extension of $\mathcal{G}$. Thus the update from $\mathcal{G}$ to $\mathcal{G}'$ is expansive gr-monotony. E is also included in the id-extension of $\mathcal{G}'$. Thus the update from $\mathcal{G}$ to $\mathcal{G}'$ is expansive id-monotony. □

Example 13. Consider AF_9 in Figure 10. $\mathcal{E}_{pr}(AF_9) = \{E_4, E_5\}$ where $E_4 = \{a, c, j\}$ and $E_5 = \{a, i\}$. AF_9^1 is an argumentation framework illustrated in Figure 11. E_4 and E_5 are also ad-sets of AF_9^1. According to Theorem 10, the update from AF_9 to AF_9^1 is expansive co and pr-monotony, while $\{a, d, j, d\}$, $\{a, d, j, e\}$, $\{a, i, d\}$ and $\{a, i, e\}$ are co and pr-extensions of AF_9^1.

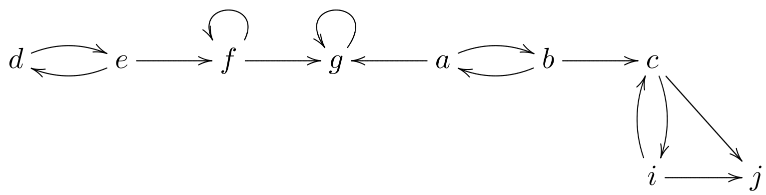
$\mathcal{E}_{gr}(AF_9) = \mathcal{E}_{id}(AF_9) = \{E_3\}$ where $E_3 = \{a\}$. AF_9^2 illustrated in Figure 12 has E_3 as the gr-extension. According to Theorem 10, the update from AF_9 to AF_9^2 is expansive gr and id-monotony, and we can see from Figure 12 that $\{a, e\}$ is the id-extension of AF_9^2.

$\mathcal{G}$ is the original argumentation framework, and $\mathcal{G}'$ is the updated framework. If the minimal co-extention, i.e. the gr-extension of $\mathcal{G}$ is restricted in $\mathcal{G}'$, then all



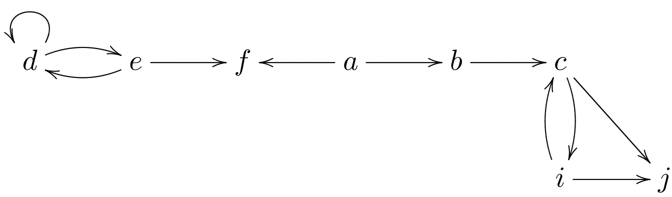
AF_9

Figure 10: An argumentation framework



AF_9^1

Figure 11: An updated argumentation framework of AF_9



AF_9^2

Figure 12: An updated argumentation framework of AF_9

co-extensions are restricted in $\mathcal{G}'$. The minimal gr, pr and id-extensions of an argumentation framework are themselves. The restrictive monotony under grounded, preferred and ideal semantics are satisfied if the corresponding gr, pr and id-extensions are restricted in $\mathcal{G}'$.

The following theorem uses the set of σ-argumentation frameworks with an extension to show some sufficient conditions for restrictive monotony.

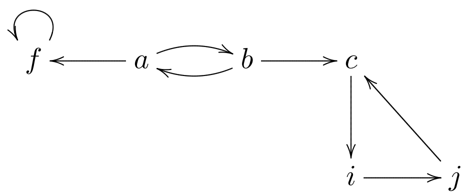
Theorem 11. *Let $\mathcal{G} = (A, R)$ and $\mathcal{G}' = (A', R')$ be argumentation frameworks.*

- *If E is the gr-extension of $\mathcal{G}$ and $\mathcal{G}' \in AF_E^{co}$, then the update from $\mathcal{G}$ to $\mathcal{G}'$ is restrictive co and gr-monotony.*
- *If E is the id-extension of $\mathcal{G}$ and $\mathcal{G}' \in AF_E^{pr}$, then the update from $\mathcal{G}$ to $\mathcal{G}'$ is restrictive pr and id-monotony.*

Proof.

- Suppose E is the gr-extension of $\mathcal{G}$ and $\mathcal{G}' \in AF_E^{co}$, then E is a co-extension of $\mathcal{G}'$. Since gr-extension is the minimal co-extension of an argumentation framework, then the update from $\mathcal{G}$ to $\mathcal{G}'$ is restrictive co-monotony. Moreover, there must be a gr-extension E' of $\mathcal{G}'$ such that $E' \subseteq E$, then the update from $\mathcal{G}$ to $\mathcal{G}'$ is also restrictive gr-monotony.
- Suppose E is the id-extension of $\mathcal{G}$ and $\mathcal{G}' \in AF_E^{pr}$, then E is a pr-extension of $\mathcal{G}'$ and it is included in any pr-extension of $\mathcal{G}$. From E is a pr-extension of $\mathcal{G}'$ we know that the id-extension E' of $\mathcal{G}'$ is included in E. Then the update from $\mathcal{G}$ to $\mathcal{G}'$ is restrictive id-monotony. From E is included in any pr-extension of $\mathcal{G}$ we know that there is a pr-extension E'' of $\mathcal{G}$ such that $E \subseteq E''$. Since $\mathcal{G}' \in AF_E^{pr}$, then the update from $\mathcal{G}$ to $\mathcal{G}'$ is restrictive pr-monotony. □

Example 14. Consider AF_9 illustrated in Figure 10 and $E_3 = \{a\}$. E_3 is the gr and the id-extension of AF_9, and it is also a co-extension and a pr-extension of AF_9^3 illustrated in Figure 13. According to Theorem 11, the update from AF_9 to AF_9^3 is restrictive co, gr, pr and id-monotony: $E_4 = \{a, c, j\}$ and $E_5 = \{a, i\}$ are co and pr-extensions of AF_9; $\emptyset$ is the gr and the id-extension of AF_9^3.



AF_9^3

Figure 13: An updated argumentation framework of AF_9

5 Conclusion

Given an argumentation framework, a set of extensions are generated from its structure. Reversely, given an extension, or a set of extensions, a set of argumentation frameworks can be decided. In this paper, we study how to characterize argumentation frameworks from an extension.

The main part of this paper is Section 3, in which we define, and characterize the argumentation framework, say $\mathcal{G}$, with a set of arguments, say E, as an extension. A series of characterizing theorems are proposed. The characterizations of $\mathcal{G}$ under complete, grounded and stable semantics are in the syntax level, but the characterization of $\mathcal{G}$ under preferred semantics is decided partially by computing the admissible labelling of a related sub-framework $\mathcal{G}\downarrow_{I^E_{\mathcal{G}}}$. The last part of Section 3 shows the relations between the sets of argumentation frameworks with an extension under different semantics, and that how to characterize an argumentation framework with more than one extension. In [17], we characterize subgraphs of an argumentation framework with an extension under admissible, complete, stable, preferred and grounded semantics[1]. In this paper, we adjust the characterizations to whole argumentation frameworks. The most significant improvement of this paper is that we found a way to characterize the argumentation framework $\mathcal{G}$ without computing the semantics of $\mathcal{G}\downarrow_{E\cup E^+_{\mathcal{G}}}$ under grounded semantics, while in [17], the counterpart of $\mathcal{G}\downarrow_{E\cup E^+_{\mathcal{G}}}$ needs to be checked whether has E as the grounded extention. Section 3 is a groundwork for [22] and [21]. All results related to revising/updating argumentation frameworks in [22] and [21] conform to the characterizing theorems.

Section 4 shows some applications of the work in Section 3. The first application is updating an argumentation framework to enforce an extension. We discuss this problem followed from Baumann et al. ([4]), and this part of application is not a new idea but a review of [21]. The second application is monotony which is a property of updating argumentation frameworks. The monotony is introduced followed from [5], and is separated as expansive monotony and restrictive monotony. There are some relations between the set of σ-arguementation frameworks with an extension and monotony, and the set of σ-arguementation frameworks with an extension is used in this paper to show some conditions for monotony. Comparing [4] and [5], where updating an arugmentation framework is just adding or deleting one argument and the related attacks ([5]), or extending it ([4]), we combine our work on characterizing theorems to the updating process and provide a discretionary way to update a framework for both enforcement and monotony.

Two conceptions are provided in this paper to help characterizing the σ-arguementation framework with an extension. The first is circle. The circles in an argumentation framework are the key points to the constitution of semantics. It is used to characterize the gr-arguementation framework with an extension. The second one is partially labeled sub-framework. It is a variation of the concept of the same name in [15], and it is used to prove the characterization of pr-arguementation framework with an extension. Furthermore, the partially labeled sub-framework itself is a useful idea to study the merging of argumentation frameworks.

To conclude, there are three defects of our work. The first is on the characterization of pr-argumentation frameworks with an extension, where computing semantics of sub-frameworks makes the process of constructing a requested argumentation framework complicated. The second is that the argumentation framework generated from the characterizing theorems may have one or more extensions undesired. We can not get a one-to-one match between the argumentation frameworks and a set of

[1]"Admissible" is a kind of semantics in [17].

extensions. The third is that we do not get the characterizing theorems under some other kinds of semantics such as ideal and semi-stable. All of these problems will be worth discussing in the future.

References

[1] P. Baroni, M. Caminada and M. Giacomin, 2011, "An introduction to argumentation semantics", *Knowledge Engineering Review*, **26(4)**: 365–410.

[2] P. Baroni, M. Giacomin and G. Guida, 2005, "SCC-recursiveness: A general schema for argumentation semantics", *Artificial Intelligence*, **168(1-2)**: 162–210.

[3] R. Baumann, 2012, "What does it take to enforce an argument? Minimal change in abstract argumentation", *Proceedings of the 20th European Conference on Artificial Intelligence*, pp. 127–132, Amsterdam: IOS Press.

[4] R. Baumann and G. Brewka, 2010, "Expanding argumentation frameworks: Enforcing and monotonicity results", *Proceedings of the 2010 conference on Computational Models of Argument*, pp. 75–86, Amsterdam: IOS Press.

[5] P. Bisquert, C. Cayrol, F. D. de Saint-Cyr and M.-C. Lagasquie-Schiex, 2013, "Characterizing change in abstract argumentation systems", in E. Fermé, D. Gabbay and G. Simari (eds.), *Trends in Belief Revision and Argumentation Dynamics*, **Vol. 48**, pp. 75–102, London: College Publications.

[6] G. Boella, S. Kaci and L. van der Torre, 2009, "Dynamics in argumentation with single extensions: Abstraction principles and the grounded extension", in S. Claudio and C. Gaetano (eds.), *Symbolic and Quantitative Approaches to Reasoning with Uncertainty*, pp. 107–118, Berlin: Springer.

[7] G. Boella, S. Kaci and L. van der Torre, 2010, "Dynamics in argumentation with single extensions: Attack refinement and the grounded extension (extended version)", *Argumentation in Multi-Agent Systems: 6th International Workshop*, pp. 150–159, Berlin: Springer.

[8] M. Caminada and G. Pigozzi, 2011, "On judgment aggregation in abstract argumentation", *Autonomous Agents and Multi-Agent Systems*, **22(1)**: 64–102.

[9] C. Cayrol, F. D. de Saint-Cyr and M.-C. Lagasquie-Schiex, 2008, "Revision of an argumentation system", *Principles of Knowledge Representation and Reasoning: Proceedings of the 11th International Conference, KR 2008*, pp. 124–134, Menlo Park, CA: AAAI Press.

[10] C. Cayrol, F. D. de Saint-Cyr and M.-C. Lagasquie-Schiex, 2010, "Change in abstract argumentation frameworks: Adding an argument", *Journal of Artificial Intelligence Research*, **38**: 49–84.

[11] P. M. Dung, 1995, "On the acceptability of arguments and its fundamental role in nonmonotonic reasoning, logic programming and n-person games", *Artificial Intelligence*, **77(2)**: 321–357.

[12] P. E. Dunne and T. Bench-Capon, 2002, "Coherence in finite argument systems", *Artificial Intelligence*, **141(1-2)**: 187–203.

[13] P. E. Dunne and M. Wooldridge, 2009, “Complexity of abstract argumentation”, in I. Rahwan and G. R. Simari (eds.), *Argumentation in Artificial Intelligence,* **Ch. 5**, pp. 85–104, Boston, MA: Springer.

[14] E. J. Kim, S. Ordyniak and S. Szeider, 2011, “Algorithms and complexity results for persuasive argumentation”, *Artificial Intelligence*, **175(9–10)**: 1722–1736.

[15] B. Liao, 2014, *Efficient Computation of Argumentation Semantics*, Zhejiang University Press.

[16] B. Liao, L. Jin and R. C. Koons, 2011, “Dynamics of argumentation systems: A division-based method”, *Artificial Intelligence*, **175(11)**: 1790–1814.

[17] B. Liao, K. Xu and H. Huang, 2018, “Formulating semantics of probabilistic argumentation by characterizing subgraphs: Theory and empirical result”, *Journal of Logic and Computation*, **28(2)**: 305–335.

[18] C. Martin, W. Carnielli and P. E. Dunne., 2012, “Semi-stable semantics”, *Journal of Logic and Computation*, **22(5)**: 1207–1254.

[19] S. Modgil and M. Caminada., 2009, “Proof theories and algorithms for abstract argumentation frameworks”, in I. Rahwan and G. R. Simari (eds.), *Argumentation in Artificial Intelligence,* **Ch. 6**, pp. 105–129, Boston, MA: Springer.

[20] M. G. Pietro Baroni, 2007, “On principle-based evaluation of extension-based argumentation semantics”, *Artificial Intelligence*, **171(10-15)**: 675–700.

[21] K. Xu and B. Liao, 2019, “Updating argumentation frameworks for enforcing extensions”, in L. Beishui, Å. Thomas and W. Yi (eds.), *Dynamics, Uncertainty and Reasoning*, pp. 63–79, Singapore: Springer Singapore.

[22] K. Xu and B. Liao, 2020, “Revision on the abstract argumentation system to enforce an extension”, *Studies in Logic*, **13(5)**: 24–47.

根据外延构建论辩框架的研究

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摘　要

抽象论辩的研究可以分为两个方向，一是依据论辩框架的结构，求取论辩语义；一是依据论辩语义，构建论辩框架。第一个方向目前已经得到了广泛的关注和研究，但是很少学者关注第二个研究方向。本文着手于论辩框架的构建及其应用：给定一个论证集合，找寻以此集合为外延的论辩框架，描述他们的结构特征，而后，将研究结果应用于抽象论辩的动态性中。

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